Reliability sensitivity analysis with dependent inputs

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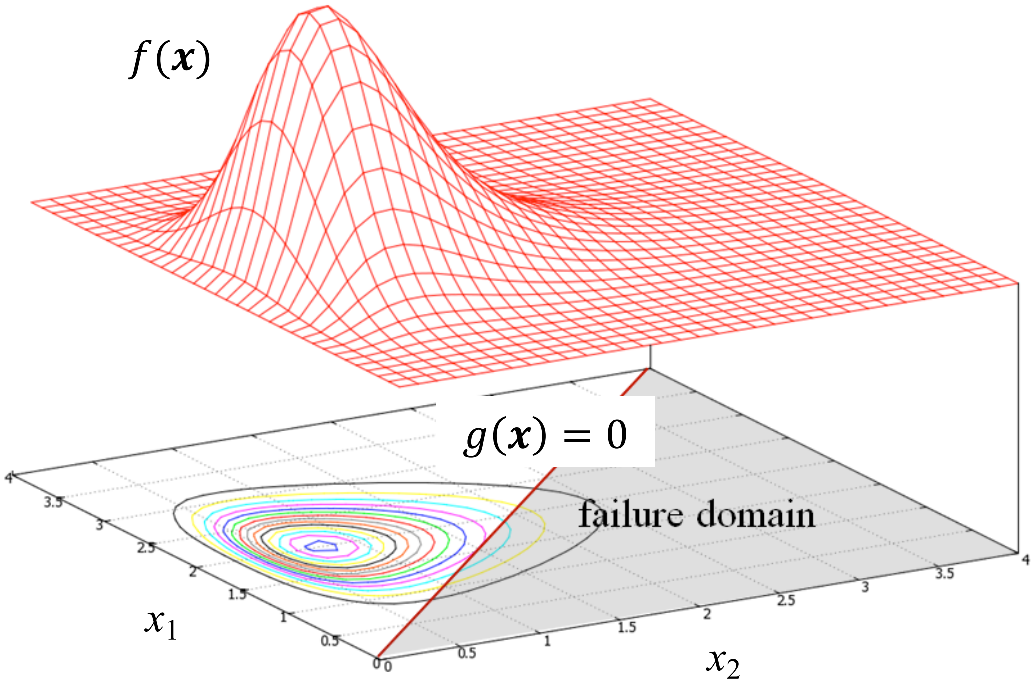
Reliability analysis

Input random variables: $\boldsymbol{X} = [X_1; X_2; \dots; X_d]$ with joint PDF $f(\boldsymbol{x})$

Limit-state function $g(\boldsymbol{x}) = \tilde{g} \circ \mathcal{M}(\boldsymbol{x})$; Failure event $F = \{\boldsymbol{x} \in \mathbb{R}^d : g(\boldsymbol{x}) \leq 0\}$

Probability of failure:

$$p_F := \mathbb{P}(F) = \int_{g(\boldsymbol{x}) \leq 0} f(\boldsymbol{x}) d\boldsymbol{x} = \mathbb{E}[I(g(\boldsymbol{X}) \leq 0)]$$



Reliability methods

- Approximation methods based on Taylor series: FORM/SORM
- Sampling methods, e.g. splitting, subset simulation, cross-entropy method, line sampling, sequential importance sampling
- Surrogate-based reliability methods, e.g. Gaussian process (kriging), polynomial chaos expansion (PCE), neural networks, low-rank tensors

Reliability sensitivity analysis

- Gradient-based sensitivity analysis: How does a change in the (deterministic) input parameters influences p_F? [Wu 1994; Jensen et al. 2009; Papaioannou et al. 2013, 2018]
- Variance-based/global sensitivity analysis: How does the variability of the input random variables influences p_F? [Li et al. 2012; Wei et al. 2012; Ehre et al. 2020]
- Decision-theoretic sensitivity analysis: How does the reduction of uncertainty of the input random variables influence the optimality of an engineering decision? [Straub et al. 2022]

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Variance-based sensitivity analysis

Consider an n-dimensional independent random vector $\boldsymbol{X}$ and a function $Q = h(\boldsymbol{X})$.

ANOVA representation:

$$h(\boldsymbol{X}) = h_0 + \sum_{i=1}^{d} h_i(X_i) + \sum_{i<j}^{d} h_{ij}\left(X_i, X_j\right) + \cdots + h_{1\dots n}(X_1, \dots, X_n)$$

The ANOVA representation exists and is unique provided that for any subset $\boldsymbol{X}_{\boldsymbol{v}} \subseteq \boldsymbol{X}$ and any $i \in \boldsymbol{v}$

$$\mathbb{E}\left[h_{\boldsymbol{v}}(\boldsymbol{X}_{\boldsymbol{v}}) | \boldsymbol{X}_{\boldsymbol{v}\setminus i}\right] = \int_{-\infty}^{\infty} h_{\boldsymbol{v}}(\boldsymbol{x}_{\boldsymbol{v}})\, f_i(x_i) dx_i = 0$$

Variance decomposition:

$$\mathbb{V}(Q) = \sum_{i=1}^{d} V_i + \sum_{i<j}^{d} V_{ij} + \cdots + V_{1\dots d}$$

Variance-based sensitivity indices

First-order (Sobol') indices:

$$S_i = \frac{V_i}{\mathbb{V}(Q)} = \frac{\mathbb{V}(\mathbb{E}[Q|X_i])}{\mathbb{V}(Q)}$$

Total effect indices

$$S_i^T = \frac{V_i + \sum_{j\neq i}^{d} V_{ij} + \cdots + V_{1\ldots d}}{\mathbb{V}(Q)} = 1 - \frac{\mathbb{V}(\mathbb{E}[Q|\boldsymbol{X}_{\sim i}])}{\mathbb{V}(Q)}$$

It holds: $0 \leq S_i \leq S_i^T \leq 1$

Estimation of variance-based sensitivity indices

- Double loop Monte Carlo procedures are computationally prohibitive
- Peak-freeze estimators with N samples: $N_T = N(d+2)$ [Sobol' 1993; Jansen 1999; Saltelli et al. 2010]
- Spectral methods: FAST, RBD, polynomial chaos [Efron & Stein 1981; Plischke 2010; Sudret 2008; Ehre et al. 2020]

Variance-based reliability sensitivities

Failure event $F = \{\boldsymbol{x} \in \mathbb{R}^d : g(\boldsymbol{x}) \leq 0\}$

Quantity of interest: $Z = I(g(\boldsymbol{X}) \leq 0)$

It is $\mathbb{E}[Z] = p_F, \mathbb{V}[Z] = p_F(1 - p_F)$

First-order indices:

$$S_{F,i} = \frac{\mathbb{V}(\mathbb{E}[Z|X_i])}{\mathbb{V}(Z)} = \frac{\mathbb{V}(\mathbb{P}[F|X_i])}{p_F(1 - p_F)}$$

Total-effect indices:

$$S_{F,i}^T = 1 - \frac{\mathbb{V}(\mathbb{E}[Z|\boldsymbol{X}_{\sim i}])}{\mathbb{V}(Z)} = 1 - \frac{\mathbb{V}(\mathbb{P}[F|\boldsymbol{X}_{\sim i}])}{p_F(1 - p_F)}$$

Interpretation of reliability sensitivities

- First-order indices can be used for factor prioritization, to determine which random variable if learned will increase the accuracy of p_F the most
- Total-effect indices can be used for factor fixing, to determine the random variables with $S_{F,i}^T \approx 0$, which if fixed will not impact the prediction of p_F

Estimation of reliability sensitivities

- Peek-freeze estimators combined with importance sampling [Wei et al. 2012]
- Estimation with failure samples [Perrin & Defaux 2019, Li et al. 2019]
- Estimation with FORM [Papaioannnou & Straub 2021]

Estimation with failure samples

Probability of failure with importance sampling:

$$p_F = \mathbb{E}_\psi[I(g(\boldsymbol{X}) \leq 0)w(\boldsymbol{X})]$$

Importance weight function: $w(\boldsymbol{x}) = \frac{f(\boldsymbol{x})}{\psi(\boldsymbol{x})}$; Importance sampling density: $\psi(\boldsymbol{x})$

Estimate of probability

$$\hat{p}_F = \widehat{\mathbb{E}}_\psi[I(g(\boldsymbol{X}) \leq 0)w(\boldsymbol{X})] = \frac{1}{N}\sum_{k=1}^{N} I(g(\boldsymbol{x}_k) \leq 0)w(\boldsymbol{x}_k), \qquad \boldsymbol{x}_k \sim \psi(\cdot)$$

Optimal (zero variance) IS density

$$\psi^*(\boldsymbol{x}) = f(\boldsymbol{x}|F) = \frac{1}{p_F} I(g(\boldsymbol{x}) \leq 0) f(\boldsymbol{x})$$

The density ψ can be estimated with adaptive sampling methods [Bucher 1988; Papaioannou et al. 2016; 2019]

Estimation with failure samples (II)

First-order indices:

$$S_{F,i} = \frac{\mathbb{V}(\mathbb{E}[Z|X_i])}{\mathbb{V}(Z)} = \frac{\mathbb{V}(\mathbb{P}[F|X_i])}{p_F(1-p_F)}$$

Bayes' rule:

$$\mathbb{P}[F|X_i = x_i] = \frac{f_i(x_i|F)\mathbb{P}(F)}{f_i(x_i)}$$

$f_{X_i}(x_i|F)$ can be estimated with kernel density estimation using (weighted) failure samples [Perrin & Defaux 2019, Li et al. 2019]

Estimation with failure samples (III)

Total-effect indices:

$$S_{F,i}^{T} = 1 - \frac{\mathbb{V}(\mathbb{E}[Z|\boldsymbol{X}_{\sim i}])}{\mathbb{V}(Z)} = 1 - \frac{\mathbb{V}(\mathbb{P}[F|\boldsymbol{X}_{\sim i}])}{p_F(1-p_F)}$$

Bayes' rule:

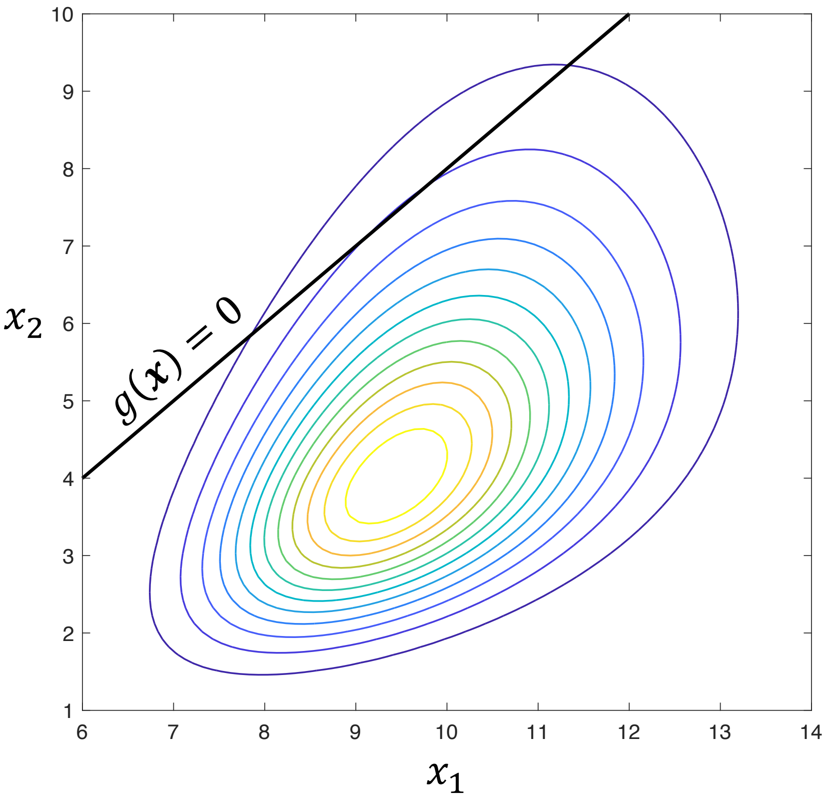
$$\mathbb{P}[F|\boldsymbol{X}_{\sim i} = \boldsymbol{x}_{\sim i}] = \frac{f_{\sim i}(\boldsymbol{x}_{\sim i}|F)\mathbb{P}(F)}{f_{\boldsymbol{x}_{\sim i}}(\boldsymbol{x}_{\sim i})}$$

$f_{\sim i}(\boldsymbol{x}_{\sim i}|F)$ can be estimated with multivariate kernel density estimation using (weighted) failure samples

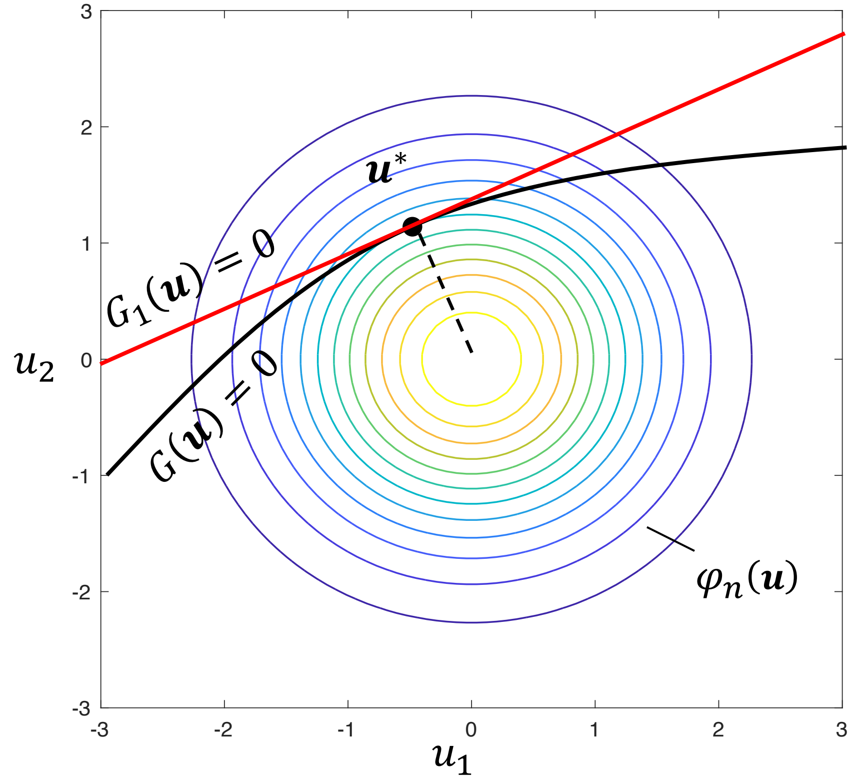
Estimation with FORM

- Transform $\boldsymbol{X}$ to an equivalent space $\boldsymbol{U} \sim N(\mathbf{0}, \mathbf{I})$, $\boldsymbol{U} = T(\boldsymbol{X})$ with $T: U_i = \Phi^{-1} \circ F_i(X_i)$
- Transformed limit-state function: $G(\boldsymbol{u}) = g \circ T^{-\mathbf{1}}(\boldsymbol{u})$
- Choose $\boldsymbol{u}^*$ as the most likely (probable) point on the hypersurface $G(\boldsymbol{u}) = 0$

Original space

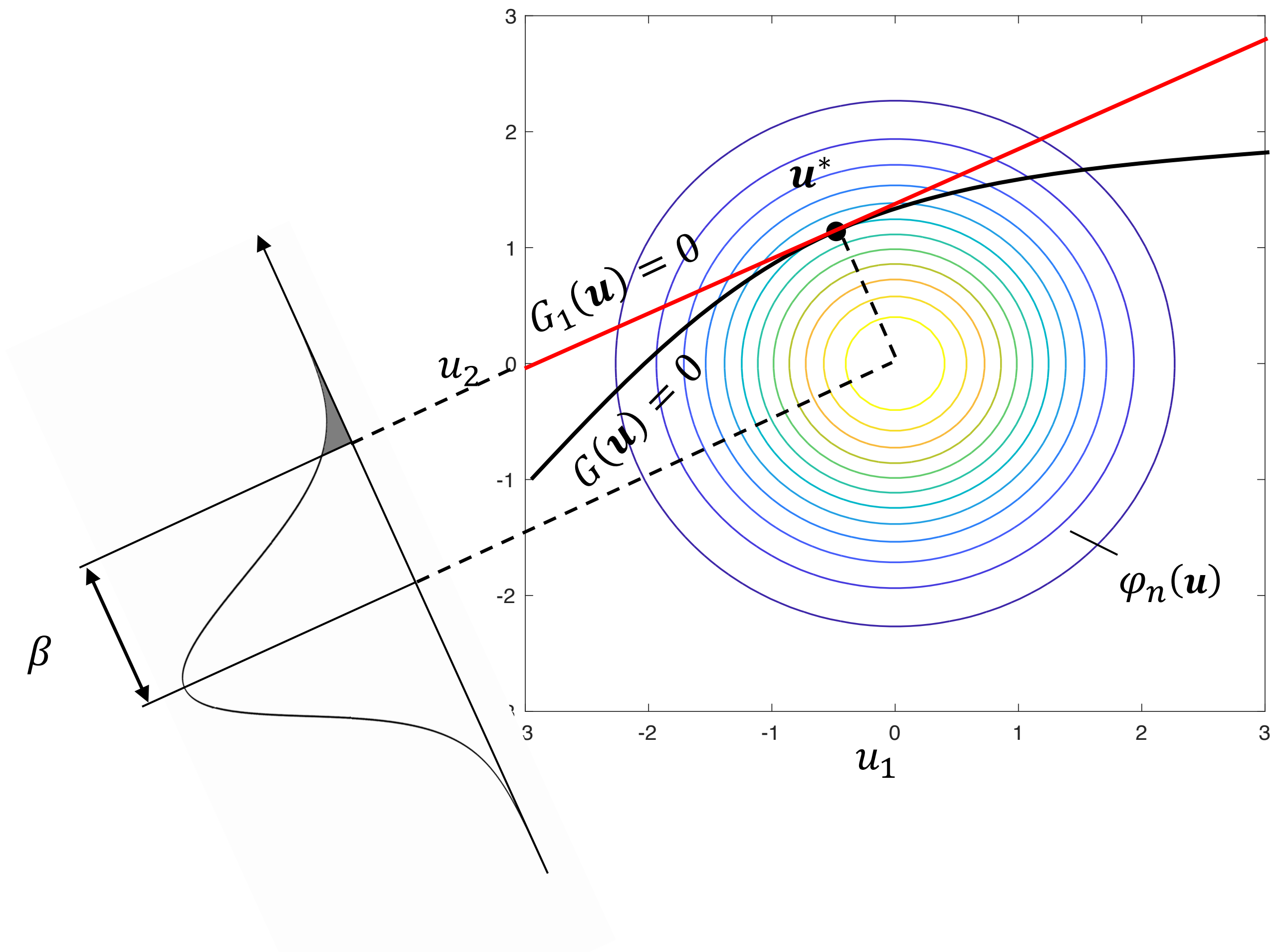


Transformed space



FORM approximation of p_F

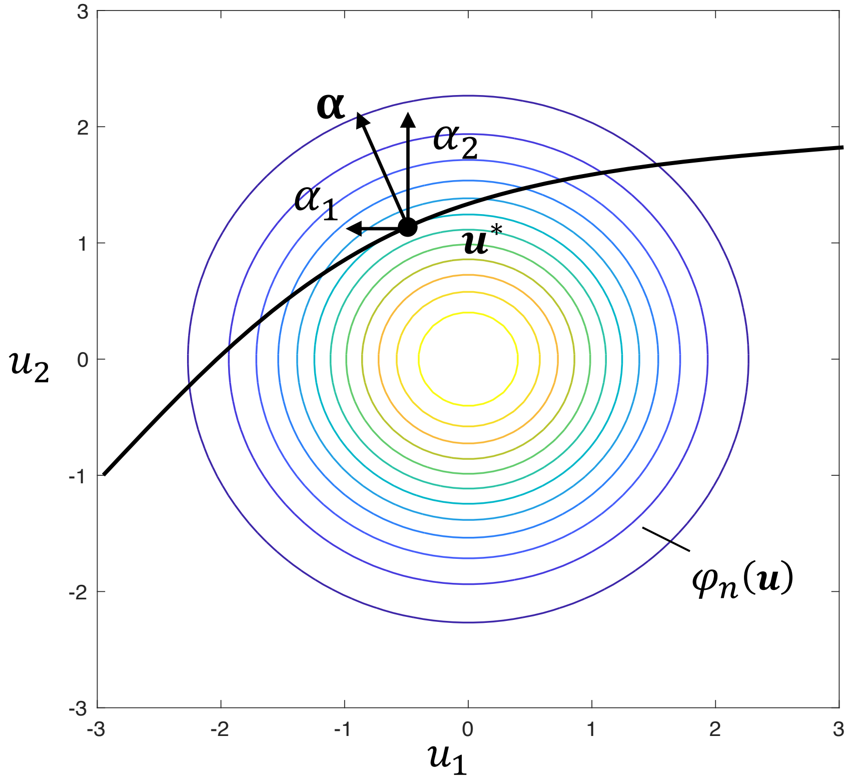
Probability approximation: $p_F \approx p_{F_1} = \Phi(-\beta)$



The FORM α-factors

Normalized negative gradient of the limit-state function at the design point $\boldsymbol{u}^*$

$$\boldsymbol{\alpha} = -\frac{\nabla G(\boldsymbol{u}^*)}{\|\nabla G(\boldsymbol{u}^*)\|} = \frac{\boldsymbol{u}^*}{\beta}$$



FORM α-factors as variance-based sensitivities

Approximation of limit-state function in $\boldsymbol{U}$-space

$$G(\boldsymbol{U}) \approx G_1(\boldsymbol{U}) = \nabla G(\boldsymbol{u}^*)(\boldsymbol{U} - \boldsymbol{u}^*) = \|\nabla G(\boldsymbol{u}^*)\|(\beta - \boldsymbol{\alpha U})$$

where $\beta = \boldsymbol{\alpha u}^*$ is the FORM reliability index

Variance decomposition of FORM approximation of the linearized limit-state function $G_1(\boldsymbol{U})$:

$$\mathbb{V}(G_1) = \|\nabla G(\boldsymbol{u}^*)\|^2 \sum_{i=1}^{n} \alpha_i^2 = \|\nabla G(\boldsymbol{u}^*)\|^2$$

First-order indices of G_1

$$S_{G_1,i} = \frac{\mathbb{V}(\mathrm{E}[G_1|U_i])}{\mathbb{V}(G_1)} = \frac{\|\nabla G(\boldsymbol{u}^*)\|^2 \alpha_i^2}{\|\nabla G(\boldsymbol{u}^*)\|^2} = \alpha_i^2$$

Variance-based SA with FORM [Papaioannou & Straub 2021]

Quantity of interest: $Z_1 = I(G_1(\boldsymbol{U}) \leq 0)$

It is $\mathbb{E}[Z_1] = p_{F_1}, \mathbb{V}[Z_1] = p_{F_1}(1 - p_{F_1})$

First-order index:

$$S_{F_1,i} = \frac{\mathbb{V}(\mathrm{E}[Z_1|U_i])}{\mathbb{V}(Z_1)} = \frac{1}{p_{F_1}\left(1 - p_{F_1}\right)} \int_0^{\alpha_i^2} \varphi_2(-\beta, -\beta, r)dr$$

Total effect index:

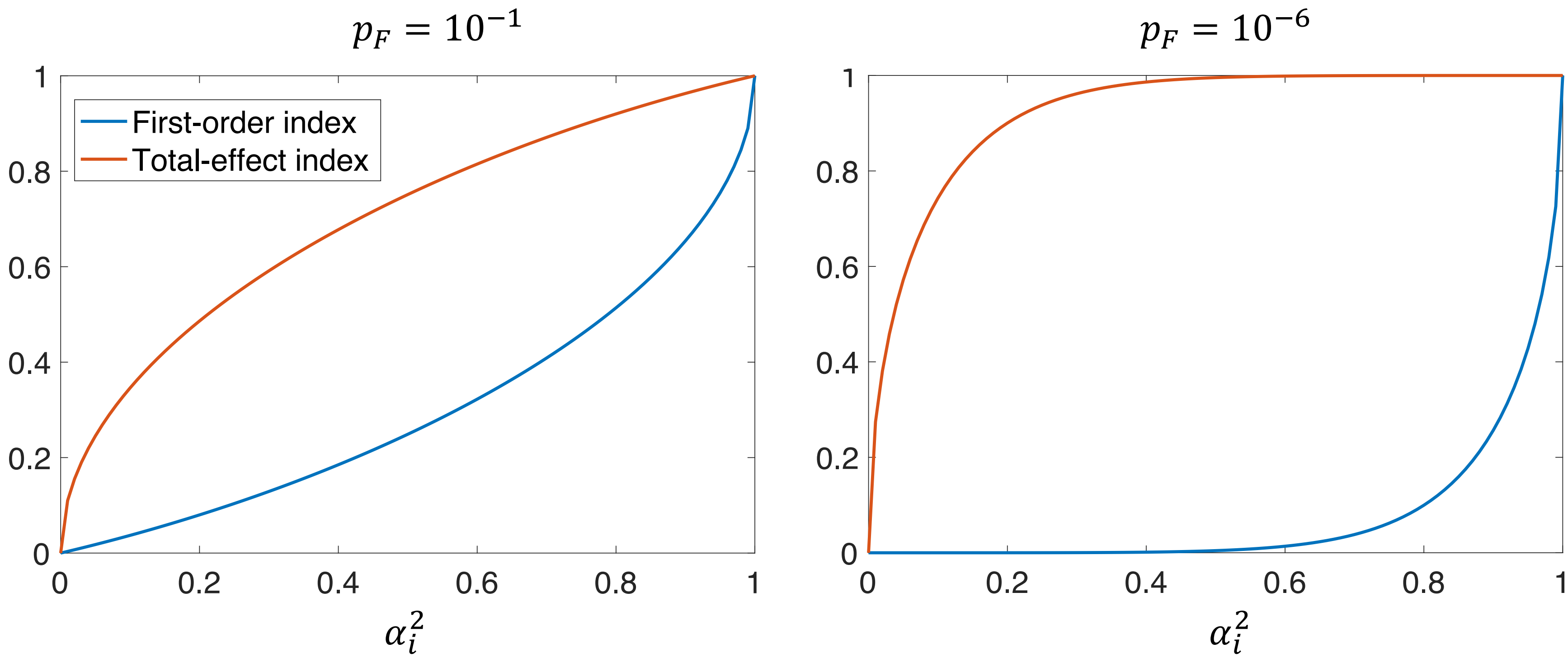
$$S_{F_1,i}^T = 1 - \frac{\mathbb{V}(\mathbb{E}[Z_1|\boldsymbol{U}_{\sim i}])}{\mathbb{V}(Z_1)} = \frac{1}{p_{F_1}\left(1 - p_{F_1}\right)} \int_{1-\alpha_i^2}^{1} \varphi_2(-\beta, -\beta, r)dr$$

with

$$\varphi_2(-\beta, -\beta, r) = \frac{1}{2\pi\sqrt{1 - r^2}} \exp\left(-\frac{\beta^2}{1 + r}\right)$$

Illustration of FORM variance-based indices

FORM-based first-order and total effect indices vs. α_i^2 for varying p_F values



Variance based SA with dependent inputs

If $\boldsymbol{X}$ is dependent…

- … the ANOVA representation is non-unique
- … the summands are not orthogonal
- … the variance decomposition includes contributions from covariance terms

Approaches for dealing with dependent inputs

- Compute terms of a generalized ANOVA decomposition [Li et al. 2010; Chastaig et al. 2012]
- Apply a transformation to decorrelate the inputs [Xu & Gertner 2008; Mara & Tarantola 2012; 2015]
- Compute Shapley effects [Owen 2014; Owen & Prieur 2017; Il Idrissi et al. 2021]

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Iso-probabilistic transform

Consider a random vector $\boldsymbol{X}: \mathcal{X} \rightarrow \mathbb{R}^d$ with joint PDF $f(\boldsymbol{x})$. Find $T: \mathbb{R}^d \rightarrow \mathbb{R}^d$ s.t.

$$\boldsymbol{U} = T(\boldsymbol{X})$$

follows the independent standard normal distribution, i.e., $\boldsymbol{U} \sim N(\mathbf{0}, \mathbf{I})$.

The transformation T is called hierarchical if it takes the following form:

$$T: \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_d \end{bmatrix} = \begin{bmatrix} \eta_1(X_1) \\ \eta_2(X_1, X_2) \\ \vdots \\ \eta_d(X_1, X_2, \dots, X_d) \end{bmatrix}$$

Example: Rosenblatt transform

$$T_R: \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_d \end{bmatrix} = \begin{bmatrix} \Phi^{-1} \circ F_1(X_1) \\ \Phi^{-1} \circ F_{2|1}(X_2|X_1) \\ \vdots \\ \Phi^{-1} \circ F_{d|1,\dots,d-1}(X_d|X_1, \dots, X_{d-1}) \end{bmatrix}$$

F_1: Marginal CDF of X_1

$F_{i|1,\dots,i-1}$: Conditional CDF of X_i given $X_1, \dots, X_{i-1}$

Example: Nataf transform

$$T_N : \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_d \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \Phi^{-1} \circ F_1(X_1) \\ \Phi^{-1} \circ F_2(X_2) \\ \vdots \\ \Phi^{-1} \circ F_d(X_d) \end{bmatrix} = \mathbf{A}^{-1}\boldsymbol{Y}$$

F_i: Marginal CDF of X_i

$\mathbf{A}$: Lower triangular matrix of the Cholesky decomposition $\mathbf{R} = \mathbf{A}\mathbf{A}^{\mathrm{T}}$

$\mathbf{R}$: Correlation matrix of $\boldsymbol{Y}$

Remarks:

- The Nataf transform implies that the vector $\boldsymbol{X}$ follows a Gaussian copula joint distribution [Lebrun & Duftoy 2009]
- The Nataf transform is equivalent to the Rosenblatt transform of the Gaussian copula model

Variance-based SA in $\boldsymbol{U}$-space

Transform the quantity of interest: $Q = h(\boldsymbol{X}) = h\big(T^{-1}(\boldsymbol{U})\big)$.

First-order (Sobol') indices:

$$S_{U_i} = \frac{\mathbb{V}(\mathbb{E}[Q|U_i])}{\mathbb{V}(Q)}$$

Total effect indices:

$$S_{U_i}^T = 1 - \frac{\mathbb{V}(\mathbb{E}[Q|\boldsymbol{U}_{\sim i}])}{\mathbb{V}(Q)}$$

Owing to the hierarchical nature of the transform:

- $S_{U_1}, S_{U_d}^T$ reflect the full first-order and total effect contributions of X_1 (including those due to its dependence with other variables)
- $S_{U_d}, S_{U_1}^T$ reflect the independent first-order and total effect contributions of X_d

Variance-based SA in $\boldsymbol{U}$-space (II)

Consider d cyclic shifts of the ordered set $\{X_1, X_2, \dots, X_d\}$.

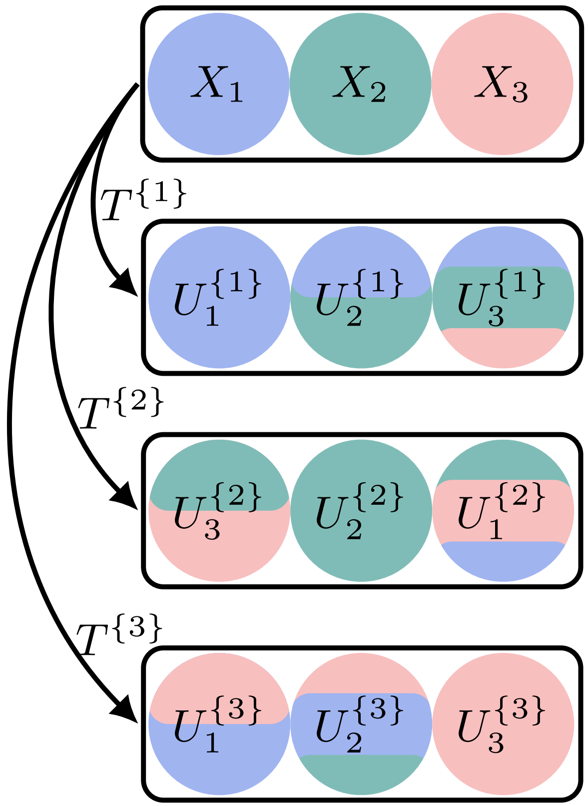
Define the set of hierarchical transforms $\left\{T^{\{i\}}\right\}_{i=1}^{d}$ with

$$T^{\{i\}}: \begin{bmatrix} U_i^{\{i\}} \\ U_{i+1}^{\{i\}} \\ \vdots \\ U_{i-1}^{\{i\}} \end{bmatrix} = \begin{bmatrix} \eta_1^{\{i\}}(X_i) \\ \eta_2^{\{i\}}(X_i, X_{i+1}) \\ \vdots \\ \eta_d^{\{i\}}(X_i, X_{i+1}, \dots, X_{i-1}) \end{bmatrix}$$

corresponding to the variable ordering $\{X_i, X_{i+1}, \dots, X_{i-1}\}$ and $X_0 \coloneqq X_d$.

Variance-based SA in $\boldsymbol{U}$-space (III)

The d cyclic shifts and corresponding hierarchical transforms allow isolating the independent contributions of each input to $\mathbb{V}(Q)$ [Mara & Tarantola 2012]



Variance-based sensitivity indices

First-order (Sobol') indices:

$$S_i = S_{U_i^{\{i\}}} = \frac{\mathbb{V}\left(\mathbb{E}\left[Q \middle| U_i^{\{i\}}\right]\right)}{\mathbb{V}(Q)}$$

$$S_{i,\mathrm{ind}} = S_{U_i^{\{i+1\}}} = \frac{\mathbb{V}\left(\mathbb{E}\left[Q \middle| U_i^{\{i+1\}}\right]\right)}{\mathbb{V}(Q)}$$

Total effect indices:

$$S_i^T = S_{U_i^{\{i+1\}}}^T = 1 - \frac{\mathbb{V}\left(\mathbb{E}\left[Q \middle| \boldsymbol{U}_{\sim i}^{\{i+1\}}\right]\right)}{\mathbb{V}(Q)}$$

$$S_{i,\mathrm{ind}}^T = S_{U_i^{\{i\}}}^T = 1 - \frac{\mathbb{V}\left(\mathbb{E}\left[Q \middle| \boldsymbol{U}_{\sim i}^{\{i\}}\right]\right)}{\mathbb{V}(Q)}$$

Peak-freeze estimators with N samples: $N_T = 0.5Nd(d+2)$ [Mara & Tarantola 2015]

Variance-based RS with dependent inputs

Failure event $F = \{\boldsymbol{x} \in \mathbb{R}^d : g(\boldsymbol{x}) \leq 0\}$

Quantity of interest: $Z = I(g(\boldsymbol{X}) \leq 0)$

First-order indices:

$$S_{F,i} = \frac{\mathbb{V}\left(\mathbb{E}\left[Z \middle| U_i^{\{i\}}\right]\right)}{\mathbb{V}(Z)} = \frac{\mathbb{V}\left(\mathbb{P}\left[F \middle| U_i^{\{i\}}\right]\right)}{p_F(1-p_F)}$$

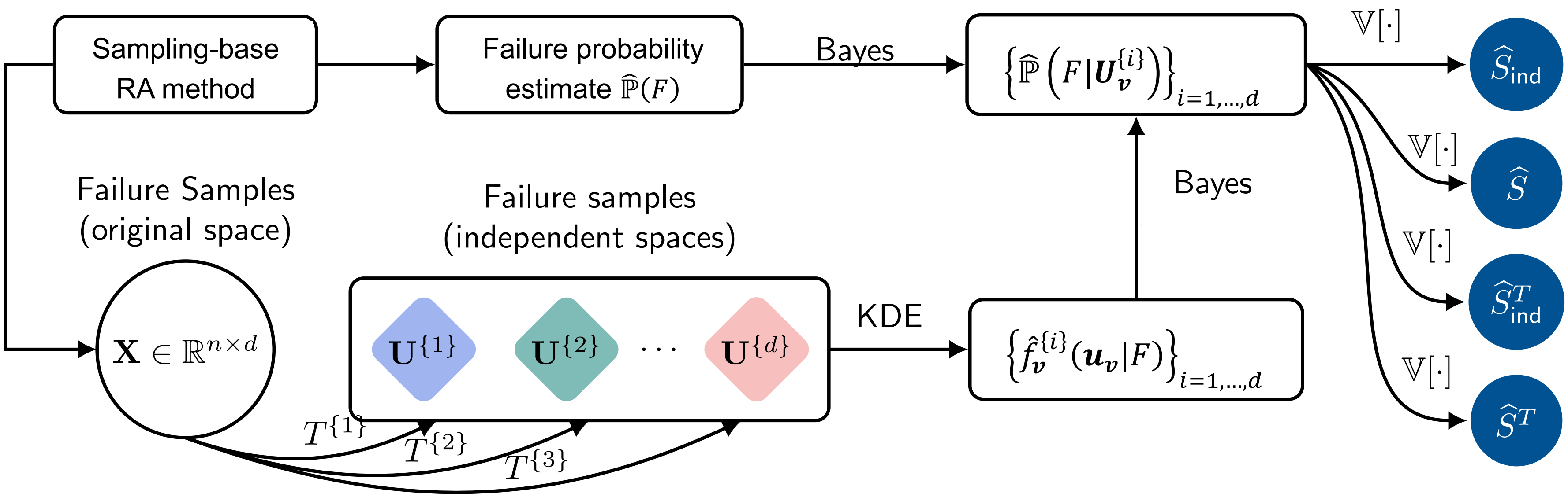
$$S_{F,i,\mathrm{ind}} = \frac{\mathbb{V}\left(\mathbb{E}\left[Z \middle| U_i^{\{i+1\}}\right]\right)}{\mathbb{V}(Z)} = \frac{\mathbb{V}\left(\mathbb{P}\left[F \middle| U_i^{\{i+1\}}\right]\right)}{p_F(1-p_F)}$$

Total-effect indices:

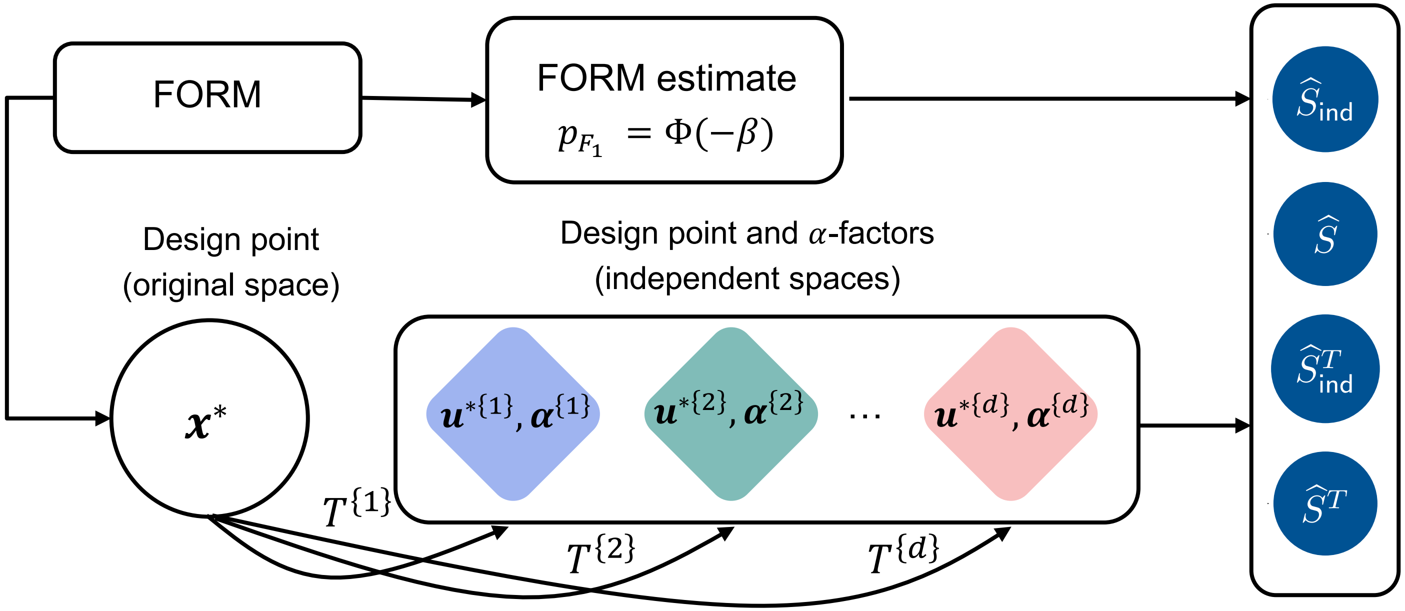
$$S_{F,i}^T = 1 - \frac{\mathbb{V}\left(\mathbb{E}\left[Z \middle| \boldsymbol{U}_{\sim i}^{\{i+1\}}\right]\right)}{\mathbb{V}(Z)} = 1 - \frac{\mathbb{V}\left(\mathbb{P}\left[F \middle| \boldsymbol{U}_{\sim i}^{\{i+1\}}\right]\right)}{p_F(1-p_F)}$$

$$S_{F,i,\mathrm{ind}}^T = 1 - \frac{\mathbb{V}\left(\mathbb{E}\left[Z \middle| \boldsymbol{U}_{\sim i}^{\{i\}}\right]\right)}{\mathbb{V}(Z)} = 1 - \frac{\mathbb{V}\left(\mathbb{P}\left[F \middle| \boldsymbol{U}_{\sim i}^{\{i\}}\right]\right)}{p_F(1-p_F)}$$

Estimation with failure samples [Ehre et al. In prep]



Estimation with FORM [Papaioannou et al. In prep]



Modified α-factors

Define:

$$\tilde{\alpha}_i = \alpha_i^{\{i\}}$$

$$\tilde{\alpha}_{i,\mathrm{ind}} = \alpha_i^{\{i+1\}}$$

The modified squared α-factors can be viewed as variance-based sensitivities in the corresponding transformed spaces:

$$\tilde{\alpha}_i^2 = S_{G_1,U_i^{\{i\}}} = \frac{\mathbb{V}\left(\mathbb{E}\left[G_1 \middle| U_i^{\{i\}}\right]\right)}{\mathbb{V}(Q)}$$

$$\tilde{\alpha}_{i,\mathrm{ind}}^2 = S_{G_1,U_i^{\{i+1\}}} = \frac{\mathbb{V}\left(\mathbb{E}\left[G_1 \middle| U_i^{\{i+1\}}\right]\right)}{\mathbb{V}(Q)}$$

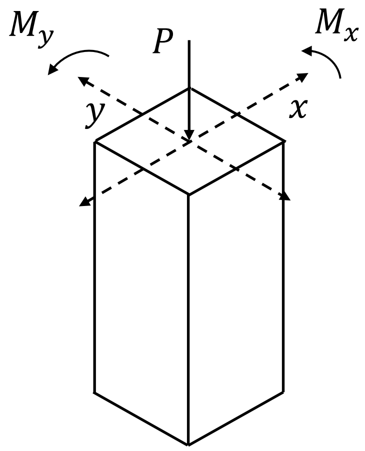
Interpretation of modified α-factors

- $\tilde{\alpha}_i^2$ is the the full first-order index of the linearized LSF transformed back to $\boldsymbol{X}$-space $G_1 \circ T(\boldsymbol{X})$
- $\tilde{\alpha}_{i,\mathrm{ind}}^2$ is the the independent first-order index of $G_1 \circ T(\boldsymbol{X})$

Remark: The γ-factors proposed in [Der Kiureghian 2005] attempt to approximate the independent factors $\tilde{\alpha}_{i,\mathrm{ind}}$ by linearizing $T(\boldsymbol{X})$.

Example: Short column

Column subjected to biaxial bending and axial force.



Limit-state function:

$$g(\boldsymbol{x}) = 1 - \frac{M_x}{s_x Y} - \frac{M_y}{s_y Y} - \left(\frac{P}{AY}\right)^2$$

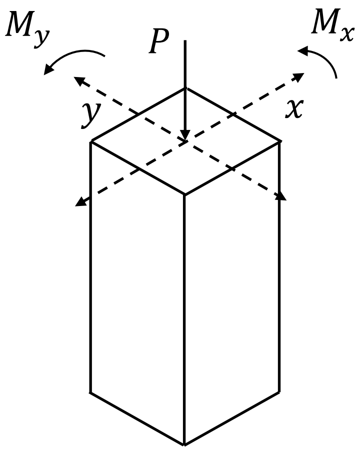
Deterministic parameters:

$s_x = 0.03\text{m}^3$, $s_y = 0.015\text{m}^3$, $A = 0.19\text{m}^2$

Parameter	Distribution	Mean	CV	$\mathbf{R}_X$			
				M_x	M_y	P	Y
M_x [KNm]	Normal	250	0.3	1.0	0.5	0.3	0.0
M_y [KNm]	Normal	125	0.3	0.5	1.0	0.3	0.0
P [kN]	Gumbel	2500	0.2	0.3	0.3	1.0	0.0
Y [MPa]	Weibull	40	0.1	0.0	0.0	0.0	1.0

Example: Short column

Column subjected to biaxial bending and axial force.



Reference probability of failure:

$p_F = 9.29\times10^{-3}$

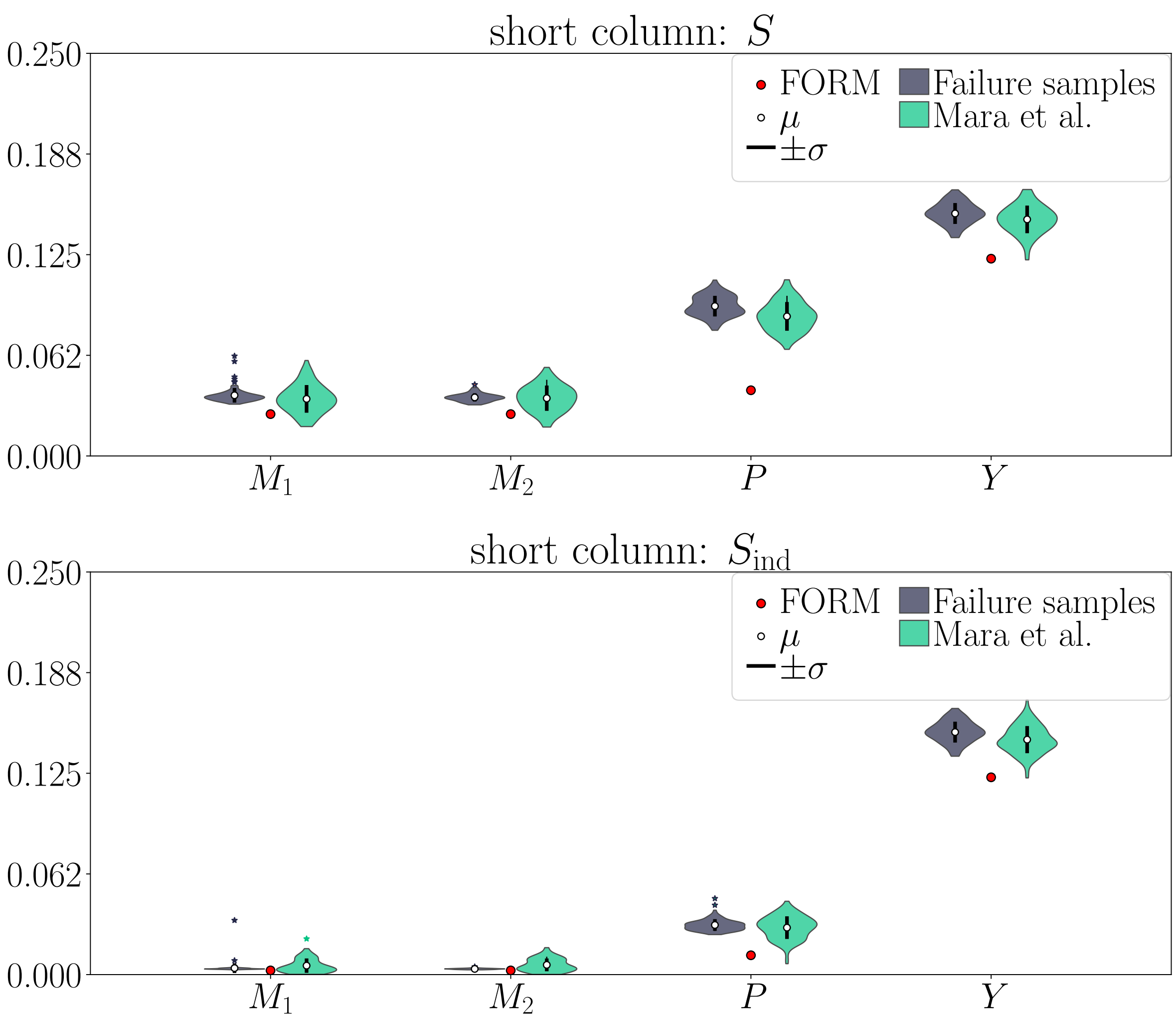
FORM estimate:

$p_F = 6.82\times10^{-3}$

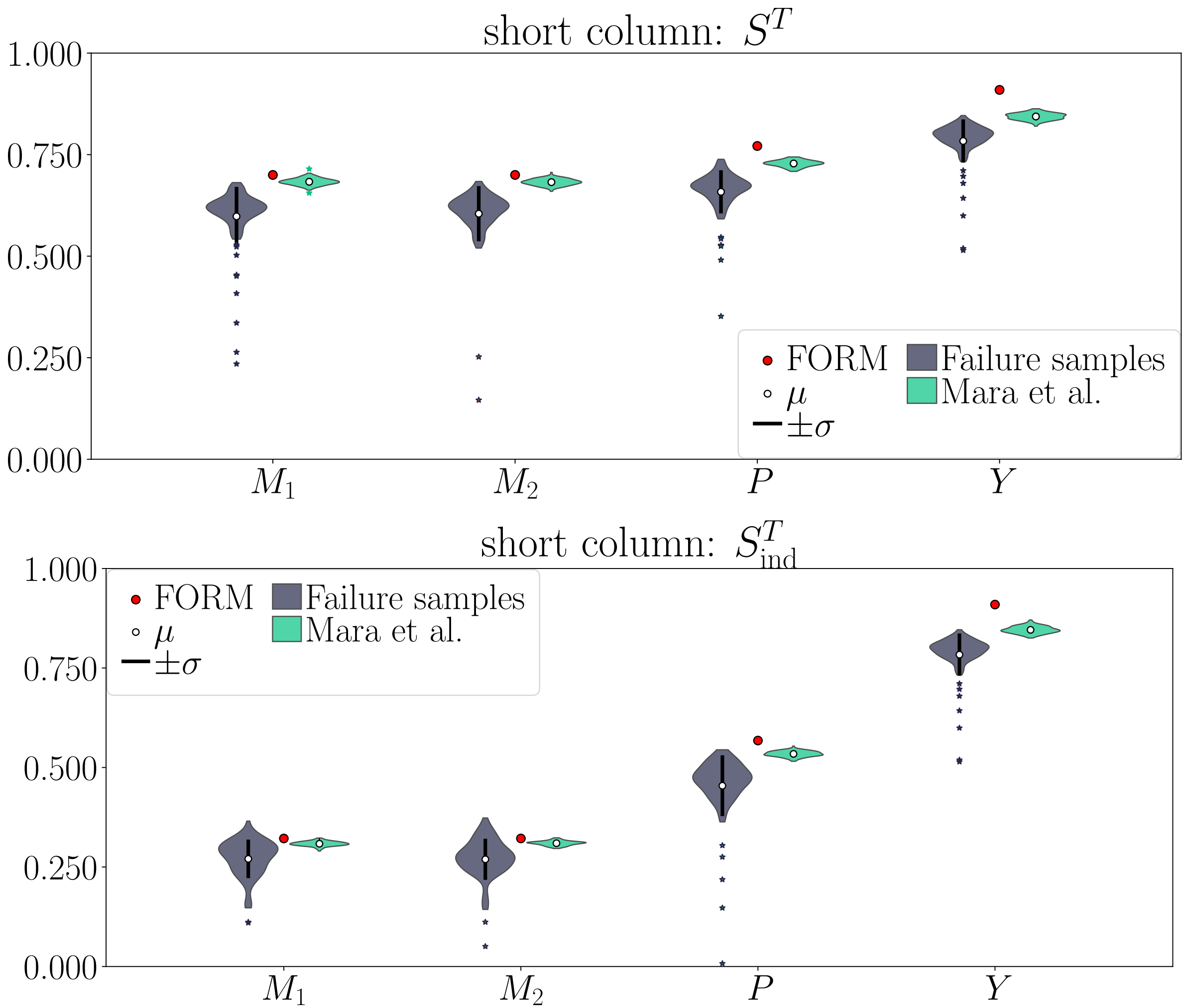
Estimates with failure samples using improved CE method [Papaioannou et al. 2019] with $N = 5 \cdot 10^4$ samples per level

Reference solution: Peak-freeze estimator with $N = 10^6$ samples [Mara & Tarantola 2015]

First-order indices



Total-effect indices



Modified α-factors

Parameter	Normalized $\tilde{\alpha}^2$	Normalized $\tilde{\alpha}_{\text{ind}}^2$	γ^2
M_x	18%	6%	7%
M_y	18%	6%	7%
P	24%	19%	20%
Y	40%	69%	65%

Conclusion

- Extension of variance-based reliability sensitivity indices for dependent inputs
- Estimates of the sensitivity indices as byproducts of standard reliability methods
- Extensions of the FORM α-factors for problems with dependent inputs
- The γ-factors of [Der Kiureghian 2005] can be viewed as approximations of the modified factors $\tilde{\alpha}_{\mathrm{ind}}$