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THE FRENCH AEROSPACE LAB

Multiphysics system analysis and optimization under uncertainty

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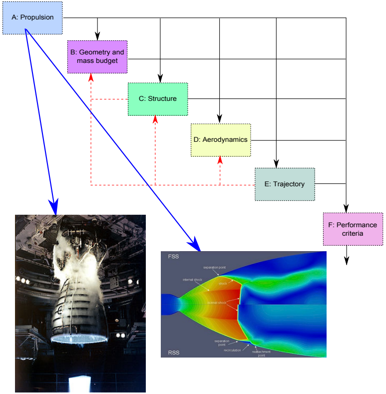
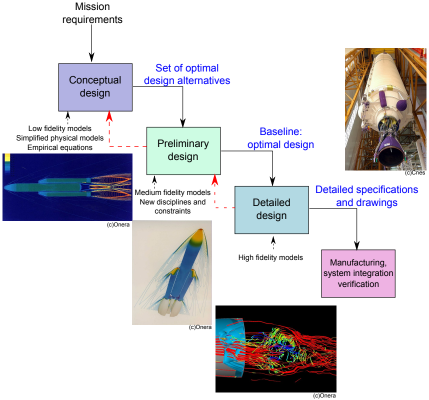
Example of multiphysics system (launcher)

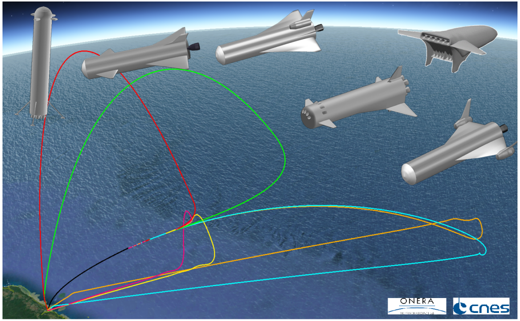




Complex process, numerous disciplines / physics

Presence of uncertainty at all the design phases

Early design of multiphysics aerospace systems

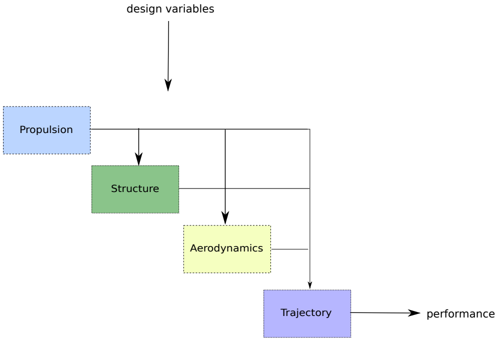


- Need to explore large number of candidate architectures at reasonable computational cost
- Need to master uncertainties relative to breakthrough technology models and environmental conditions to compare the solutions and assess trade-off

Outline

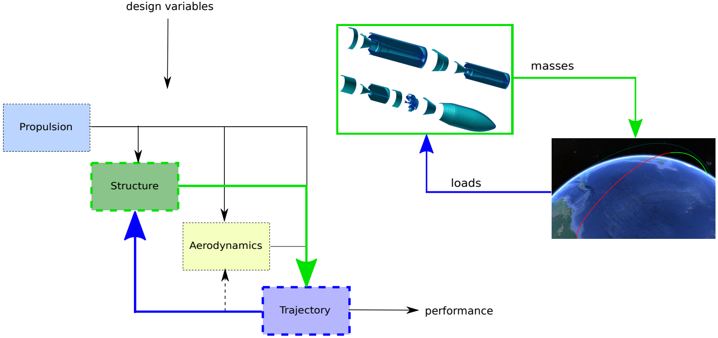
Outline

Modeling of multiphysics system



System with *feedforward* couplings

Modeling of multiphysics system

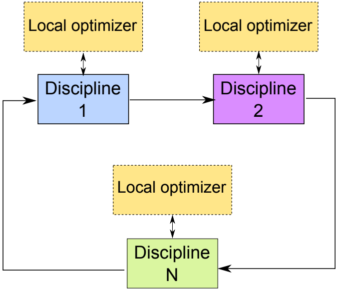


System with *feedforward and feedback* couplings

Classical approach vs MDAO

Classical design process

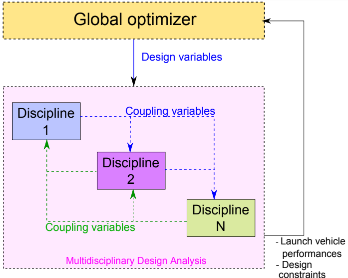
Loop between **disciplinary local optimizations**



- Risk to find local optima,
- Difficulty to meet compromises between antagonistic disciplinary objectives.

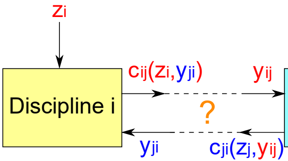
Multidisciplinary approach

Global design including all the disciplines and their interactions

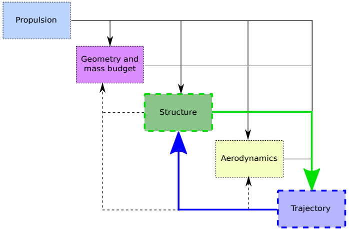


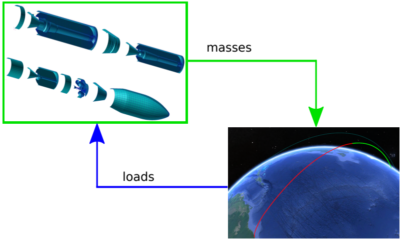
- Integration of interdisciplinary couplings in the optimization process
- Increase of optimization problem complexity

Mathematical modeling of coupled multiphysics problem

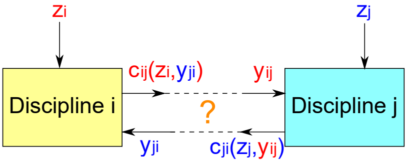


- **z** : design variables
- **y** : input coupling variables
- **c** : output coupling functions





Mathematical modeling of coupled multiphysics problem



Coupling satisfaction

$$\begin{cases} \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{ji}) \\ \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{z}_j, \mathbf{y}_{ij}) \end{cases}$$

- **z** : design variables
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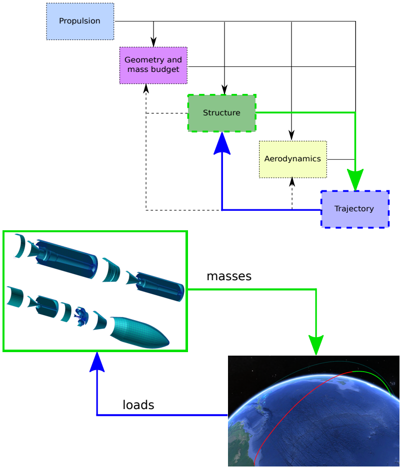
System of non linear equations



Mathematical modeling of MDAO problem

Simplified design problem

$$\begin{aligned} \text{minimize} \quad & \mathbf{f}(\mathbf{y},\mathbf{z}) \\ \text{with respect to} \quad & \mathbf{y},\mathbf{z} \\ \text{subject to} \quad & \mathbf{g}(\mathbf{y},\mathbf{z}) \leq 0 \\ & \forall j \neq i,\ \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{y}_{.j},\mathbf{z}_j) \end{aligned}$$

- **y** : coupling variables
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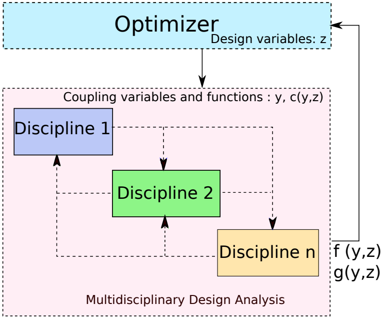
- **c** : coupling functions
- **f** : objective function
- **g** : inequality constraints

[Balesdent et al., 2012, Martins and Lambe, 2013]



Mathematical modeling of MDAO problem

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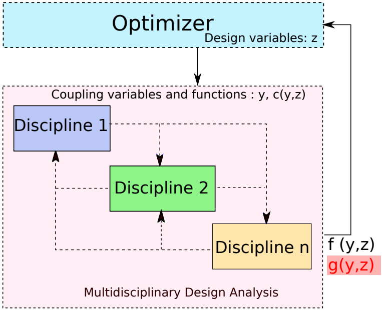

Objectives :

1. Feasibility of design problem

Mathematical modeling of MDAO problem

Simplified design problem

$$\begin{aligned} \text{minimize} \quad & \mathbf{f}(\mathbf{y}, \mathbf{z}) \\ \text{with respect to} \quad & \mathbf{y}, \mathbf{z} \\ \text{subject to} \quad & \mathbf{g}(\mathbf{y}, \mathbf{z}) \leq 0 \\ & \forall j \neq i, \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{y}_{\cdot j}, \mathbf{z}_j) \end{aligned}$$

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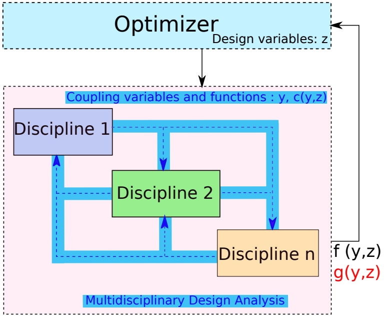

Objectives :

1. Feasibility of design problem
2. Coupling consistency

Mathematical modeling of MDAO problem

Simplified design problem

$$\begin{aligned} \text{minimize} \quad & \mathbf{f}(\mathbf{y}, \mathbf{z}) \\ \text{with respect to} \quad & \mathbf{y}, \mathbf{z} \\ \text{subject to} \quad & \mathbf{g}(\mathbf{y}, \mathbf{z}) \leq 0 \\ & \forall j \neq i, \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{y}_{\cdot j}, \mathbf{z}_j) \end{aligned}$$

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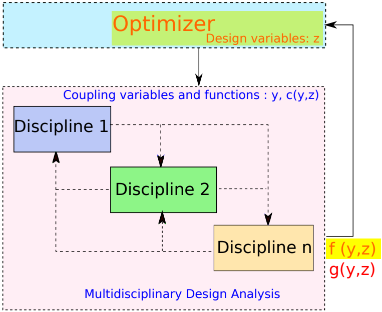
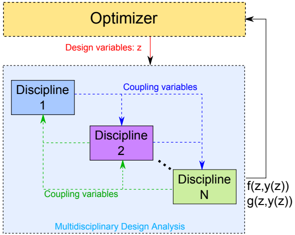


Objectives :

1. Feasibility of design problem
2. Coupling consistency
3. Optimality of optimization problem

Coupled and decoupled formulations

Coupled formulation

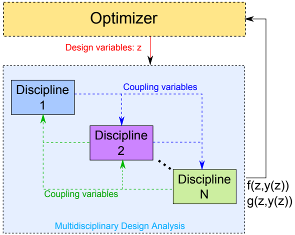


Couplings

- Interdisciplinary couplings satisfied at each iteration of the optimization process by an auxiliary solver (Gauss-Seidel, Newton, etc.)

Coupled and decoupled formulations

Coupled formulation

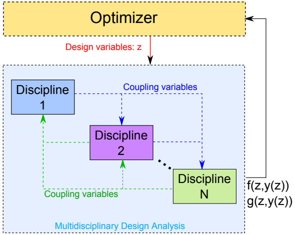


Optimization problem

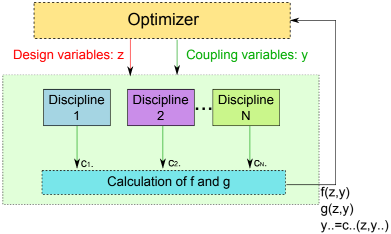
$$\begin{aligned} \text{min} \quad & f(\mathbf{z}, \mathbf{y}(\mathbf{z})) \\ \text{w.r.t} \quad & \mathbf{z} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{z}, \mathbf{y}(\mathbf{z})) \leq 0 \end{aligned}$$

Coupled and decoupled formulations [Cramer et al., 1994]

Coupled formulation - MDF (MultiDiscipline Feasible)



Decoupled formulation - IDF (Individual Discipline Feasible)



Optimization problem

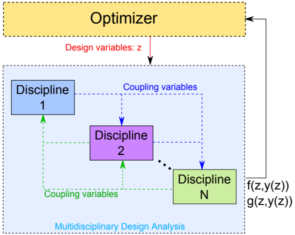
$$\begin{aligned} \min \quad & f(\mathbf{z}, \mathbf{y}(\mathbf{z})) \\ \text{w.r.t} \quad & \mathbf{z} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{z}, \mathbf{y}(\mathbf{z})) \leq 0 \end{aligned}$$

Optimization problem

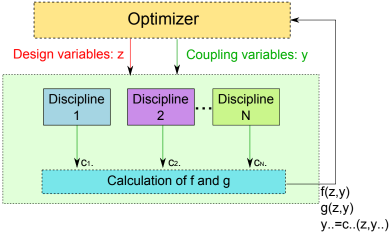
$$\begin{aligned} \min \quad & f(\mathbf{z}, \mathbf{y}) \\ \text{w.r.t.} \quad & \mathbf{z}, \mathbf{y} \\ \text{s.t.} \quad & \mathbf{g}(\mathbf{z}, \mathbf{y}) \leq 0 \\ & \forall (i,j) \in \{1, \ldots, N\}^2, i \neq j, \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i}) \end{aligned}$$

Coupled and decoupled formulations

Coupled formulation



Decoupled formulation



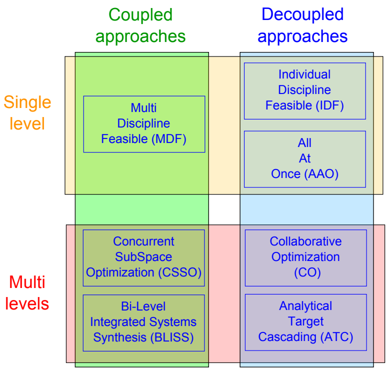
Couplings

- Interdisciplinary couplings satisfied at each iteration of the optimization process by an auxiliary solver (MultiDisciplinary Analysis - MDA)

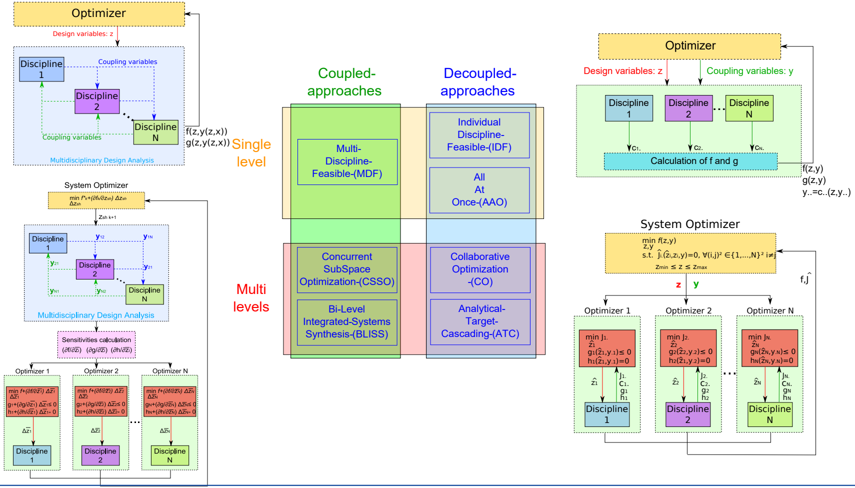
Couplings

- **Couplings variables** handled by the optimization process
- Equality constraints satisfied at the convergence of the optimization

Single-level vs Multilevel formulations



Single-level vs Multilevel formulations



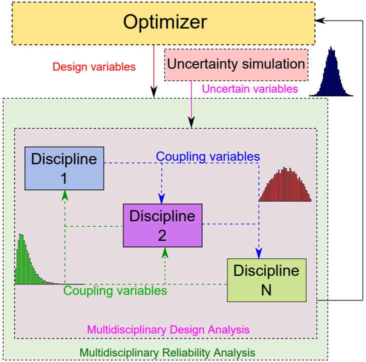
Outline

UQ for multidisciplinary systems

For multidisciplinary system analysis and optimization, individual discipline uncertainties are combined making the system analysis and optimization more complex [Brevault et al., 2020].

Three nested loops :

- multidisciplinary analysis,
- uncertainty propagation,
- optimization.



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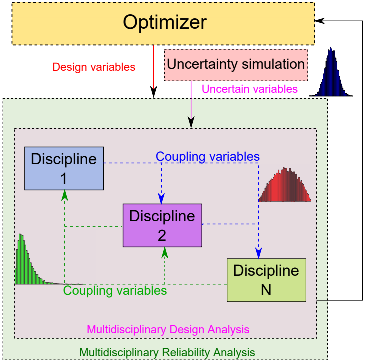
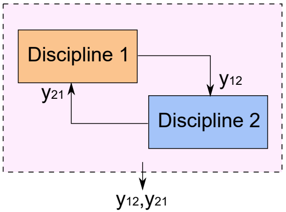


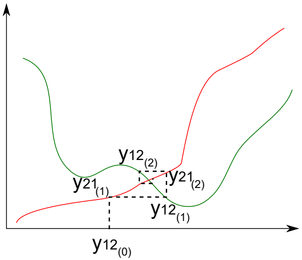
Three nested loops :

- multidisciplinary analysis,
- uncertainty propagation,
- optimization.

Need for methods to handle the numerical cost involved by these loops

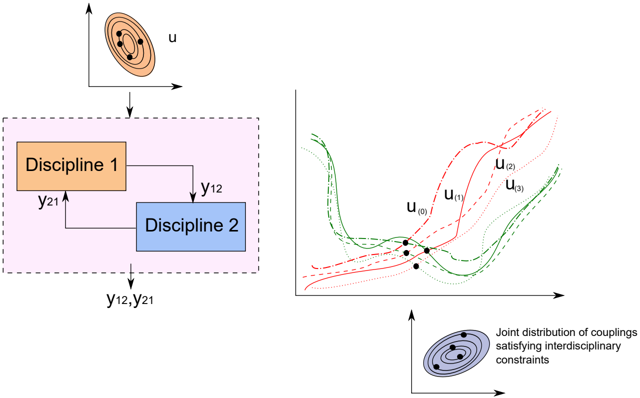
UQ for multidisciplinary systems





UQ for multidisciplinary systems

Coupling solving for system under uncertainty



MDAO vs UMDAO (Uncertainty-based MDAO)

MDAO

min
wrt
st

UMDAO

min
wrt
st

- **y** : coupling variables
- **z** : design variables
- **c** : coupling functions
- **f** : objective function
- **g** : inequality constraints

- **U** : uncertain variables

MDAO vs UMDAO (Uncertainty-based MDAO)

MDAO

min
wrt y, z
st

UMDAO

min
wrt Θ[Y], z
st

- y : coupling variables
- z : design variables
- c : coupling functions
- f : objective function
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- U : uncertain variables
- Θ[Y] : coupling variables

MDAO vs UMDAO (Uncertainty-based MDAO)

MDAO

min $\mathbf{f}(\mathbf{y}, \mathbf{z})$
wrt $\mathbf{y}, \mathbf{z}$
st

UMDAO

min $\Xi[\mathbf{f}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})]$
wrt $\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}$
st

- $\mathbf{y}$: coupling variables
- $\mathbf{z}$: design variables
- $\mathbf{c}$: coupling functions
- $\mathbf{f}$: objective function
- $\mathbf{g}$: inequality constraints

- $\mathbf{U}$: uncertain variables
- $\mathbf{\Theta}[\mathbf{Y}]$: coupling variables
- $\Xi[\cdot]$: measure of uncertainty on the objective function

- worst-case : $\min_{\mathbf{\Theta}[\mathbf{Y}],\mathbf{z}} \max_{\mathbf{U}} \mathbf{f}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})$
- robust-based formulation : $\mathbb{E}[\mathbf{f}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})] + k\sigma[\mathbf{f}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})]$,
- reliability-based formulation : $\mathbb{P}(\mathbf{g}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U}) \leq 0)$

MDAO vs UMDAO (Uncertainty-based MDAO)

MDAO

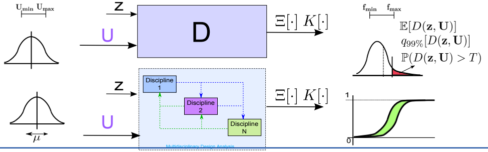
$$\begin{aligned} \min \quad & \mathbf{f}(\mathbf{y}, \mathbf{z}) \\ \text{wrt} \quad & \mathbf{y}, \mathbf{z} \\ \text{st} \quad & \mathbf{g}(\mathbf{y}, \mathbf{z}) \leq 0 \end{aligned}$$

UMDAO

$$\begin{aligned} \min \quad & \Xi[\mathbf{f}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})] \\ \text{wrt} \quad & \mathbf{\Theta}[\mathbf{Y}], \mathbf{z} \\ \text{st} \quad & \mathbf{K_g}[\mathbf{g}(\mathbf{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})] \leq 0 \end{aligned}$$

- $\mathbf{y}$: coupling variables
- $\mathbf{z}$: design variables
- $\mathbf{c}$: coupling functions
- $\mathbf{f}$: objective function
- $\mathbf{g}$: inequality constraints

- $\mathbf{U}$: uncertain variables
- $\mathbf{\Theta}[\mathbf{Y}]$: coupling variables
- $\Xi[\cdot]$: measure of uncertainty on the objective function
- $\mathbf{K}_{.}[\cdot]$: measure of constraint satisfaction under uncertainty



MDAO vs UMDAO (Uncertainty-based MDAO)

MDAO

$$\begin{array}{rc} \text{min} & \mathbf{f}(\mathbf{y}, \mathbf{z}) \\ \text{wrt} & \mathbf{y}, \mathbf{z} \\ \text{st} & \mathbf{g}(\mathbf{y}, \mathbf{z}) \leq 0 \\ & \forall j \neq i,\ \mathbf{y}_{ji} = \mathbf{c}_{ji}(\mathbf{y}_{.j}, \mathbf{z}_j) \end{array}$$

- $\mathbf{y}$: coupling variables
- $\mathbf{z}$: design variables
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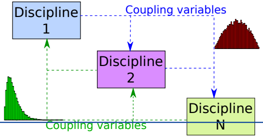
UMDAO

$$\begin{array}{rc} \text{min} & \Xi[\mathbf{f}(\boldsymbol{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})] \\ \text{wrt} & \boldsymbol{\Theta}[\mathbf{Y}], \mathbf{z} \\ \text{st} & \mathbf{K}_{\mathbf{g}}[\mathbf{g}(\boldsymbol{\Theta}[\mathbf{Y}], \mathbf{z}, \mathbf{U})] \leq 0 \\ & \forall j \neq i,\ \mathbf{K}_{\mathbf{y}}[\mathbf{Y}_{ji}(\boldsymbol{\Theta}[\mathbf{Y}]) = \mathbf{c}_{ji}(\boldsymbol{\Theta}[\mathbf{Y}_{.j}], \mathbf{z}_j, \mathbf{U})] \leq 0 \end{array}$$

- $\mathbf{U}$: uncertain variables
- $\boldsymbol{\Theta}[\mathbf{Y}]$: coupling variables
- $\Xi[\cdot]$: measure of uncertainty on the objective function [Lelièvre et al., 2016]
- $\mathbf{K}_.[\cdot]$: measure of constraint satisfaction under uncertainty



MDAO vs UMDAO

MDAO

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UMDAO

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- $\mathbf{y}$: coupling variables
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Key point : Which measure to take for the coupling satisfaction under uncertainties ?

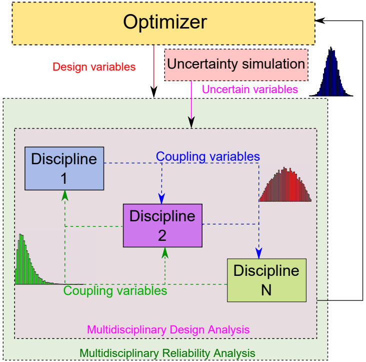
$\rightarrow \forall j \neq i, \mathbf{K}_\mathbf{y}[\mathbf{Y}_{ji}(\mathbf{\Theta}[\mathbf{Y}]) = \mathbf{c}_{ji}(\mathbf{\Theta}[\mathbf{Y}_{.j}], \mathbf{z}_j, \mathbf{U})] \leq 0$

- distribution or statistical moments,
- at a specific realization,
- for all the realizations.

MultiDiscipline Feasible with uncertainty

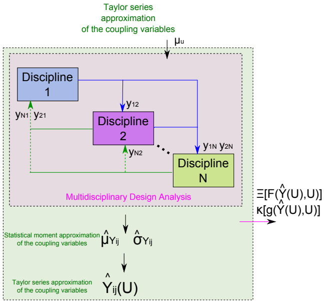
$$\begin{array}{ll} \text{min} & \Xi\left[f(\mathbf{z}, \mathbf{Y}(\mathbf{z}, \mathbf{U}), \mathbf{U})\right] \\ \text{w.r.t.} & \mathbf{z} \\ \text{s.t.} & \mathbb{K}_k\left[g_k(\mathbf{z}, \mathbf{Y}(\mathbf{z}, \mathbf{U}), \mathbf{U})\right] \leq 0 \quad \forall k \end{array}$$

- Most basic optimization framework
- Excessively expensive : three embedded loops : MDA inside UQ inside optimization [Haldar and Mahadevan, 2000, Jaeger et al., 2013]



Concurrent SubSystem Uncertainty Analysis

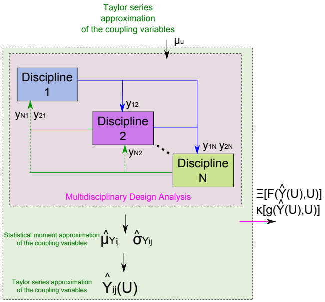
1. Full MDA at the mean value of the uncertainties $\boldsymbol{\mu}_{\mathbf{U}}$ to get $\boldsymbol{\mu}_{\mathbf{Y}_{ij}}$.



Concurrent SubSystem Uncertainty Analysis

1. Full MDA at the mean value of the uncertainties $\boldsymbol{\mu}_{\mathbf{U}}$ to get $\boldsymbol{\mu}_{\mathbf{Y}_{ij}}$.
2. Approximate the coupling variables by their first order Taylor expansion [Du and Chen, 2002] :

$$\hat{\mathbf{Y}}_{ij}\left(\mathbf{z}, \mathbf{Y}_{.i}, \mathbf{u}_i\right) = \boldsymbol{\mu}_{\mathbf{Y}_{ij}} + \left.\frac{\partial \mathbf{c}_{ij}}{\partial \mathbf{u}_i}\right|_{\mathbf{u}=\boldsymbol{\mu}_{\mathbf{u}}} \left(\mathbf{u}_i - \boldsymbol{\mu}_{\mathbf{u}_i}\right) + \left.\frac{\partial \mathbf{c}_{ij}}{\partial \mathbf{Y}_{.i}}\right|_{\mathbf{u}=\boldsymbol{\mu}_{\mathbf{u}}} \left(\mathbf{Y}_{.i} - \boldsymbol{\mu}_{\mathbf{Y}_{.i}}\right)$$



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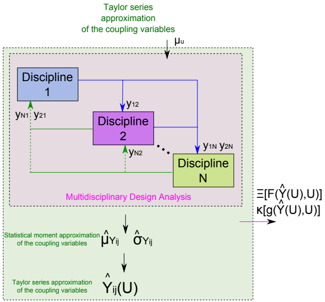
3. Replace MDA by the linearized system of equations :

$$\hat{\mathbf{Y}}(\mathbf{u}) = \mathbf{A}^{-1}\mathbf{B}(\mathbf{U} - \boldsymbol{\mu}_{\mathbf{U}})$$



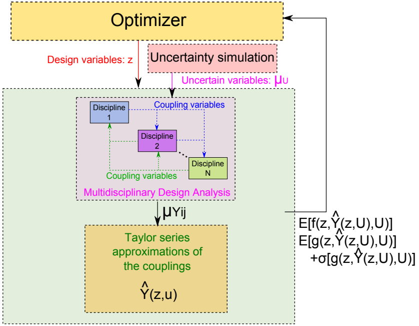
$$\mathbf{A} = \left[\left.\frac{\partial \mathbf{c}_{ij}}{\partial \mathbf{Y}_{.i}}\right|_{\mathbf{u}=\boldsymbol{\mu}_{\mathbf{U}}} \right], \quad \mathbf{B} = \left[\left.\frac{\partial \mathbf{c}_{ij}}{\partial \mathbf{u}_i}\right|_{\mathbf{u}=\boldsymbol{\mu}_{\mathbf{U}}} \right]$$

Concurrent SubSystem Uncertainty Analysis

Replace MDA by the linearized system of equations :

$$\hat{\mathbf{Y}}(\mathbf{u}) = \mathbf{A}^{-1}\mathbf{B}(\mathbf{U} - \boldsymbol{\mu}_{\mathbf{U}})$$

In case robust problem formulation is used $\Xi[\cdot] = \mathbb{E}[\cdot] + k\sigma[\cdot]$, mean value and variance of output measure can be also derived analytically model using hypothesis of linearity and independent (Normal) uncertainty.
[Du and Chen, 2002]



Concurrent SubSystem Uncertainty Analysis - Illustration

Let consider the following two discipline system [Brevault et al., 2020] :

- Discipline 1 : $y_{12} = -z_{sh}^{0.2} + u_{sh} + 0.25 \times u_1^{0.2} + z_1 + y_{21}^{0.58} + u_1^{0.4} \times y_{21}^{0.47}$
- Discipline 2 : $y_{21} = -z_{sh} + u_{sh}^{0.1} - z_2^{0.1} + 3 \times y_{12}^{0.47} + u_2^{0.33} + y_{12}^{0.16} \times u_2^{0.05} + y_{12}^{0.6} \times u_2^{0.13} + 100$

Concurrent SubSystem Uncertainty Analysis - Illustration

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$$
\begin{aligned}
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\text{- Discipline 2 : } y_{21} &= -z_{sh} + u_{sh}^{0.1} - z_2^{0.1} + 3 \times y_{12}^{0.47} + u_2^{0.33} \\
&\quad + y_{12}^{0.16} \times u_2^{0.05} + y_{12}^{0.6} \times u_2^{0.13} + 100
\end{aligned}
$$

and the objective and constraint functions :

$$
\begin{aligned}
\text{Objective : } f &= \left(\frac{1}{5}\right) \times \left[(z_{sh}-4)^2 + (z_1-3)^2 + (z_2-2)^2 + (y_{21} + z_1 \times z_2)^{0.6} + (u_{sh}+0.9)^2\right] \\
\text{Constraint : } g &= \exp(-0.01 \times u_1^2) \times (z_{sh}-1) \times z_1 - 0.02 \times u_2^5 \times z_2^3 + 0.01 \times y_{12}^{2.5} \times z_2 \times \exp(-0.1 \times u_{sh})
\end{aligned}
$$

Concurrent SubSystem Uncertainty Analysis - Illustration

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$$\begin{aligned}\text{- Discipline 1 : } y_{12} &= -z_{sh}^{0.2} + u_{sh} + 0.25 \times u_1^{0.2} + z_1 + y_{21}^{0.58} + u_1^{0.4} \times y_{21}^{0.47} \\ \text{- Discipline 2 : } y_{21} &= -z_{sh} + u_{sh}^{0.1} - z_2^{0.1} + 3 \times y_{12}^{0.47} + u_2^{0.33} \\ &\quad + y_{12}^{0.16} \times u_2^{0.05} + y_{12}^{0.6} \times u_2^{0.13} + 100\end{aligned}$$

and the objective and constraint functions :

$$\text{Objective : } f = \left(\frac{1}{5}\right) \times \left[(z_{sh} - 4)^2 + (z_1 - 3)^2 + (z_2 - 2)^2 + (y_{21} + z_1 \times z_2)^{0.6} + (u_{sh} + 0.9)^2\right]$$

$$\text{Constraint : } g = \exp(-0.01 \times u_1^2) \times (z_{sh} - 1) \times z_1 - 0.02 \times u_2^5 \times z_2^3 + 0.01 \times y_{12}^{2.5} \times z_2 \times \exp(-0.1 \times u_{sh})$$

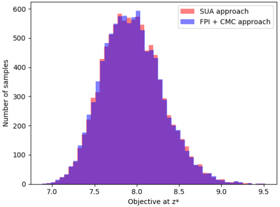
- $U_{sh} \sim \mathcal{N}(2, 0.3)$
- $U_1 \sim \mathcal{N}(0.5, 0.1)$
- $U_2 \sim \mathcal{N}(4, 0.5)$

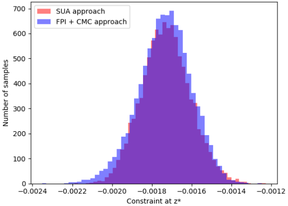
Reference solution obtained by MDF with CMC :

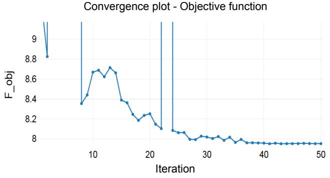
- $\mathbf{z} = [0.6; 0.2; 0.0]^T$
- Objective function : 7.95

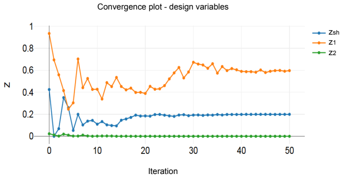
Concurrent SubSystem Uncertainty Analysis - Illustration

Convergence to the same design point as MDF with a reduction factor of 10^3 : $N_{opt} \times N_{MDA} \times N_{CMC} \rightarrow N_{opt}$









Concurrent SubSystem Uncertainty Analysis - Illustration

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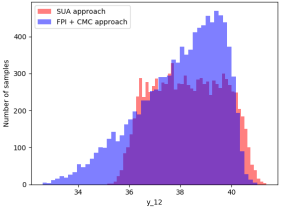
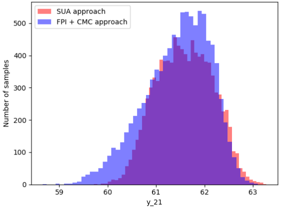


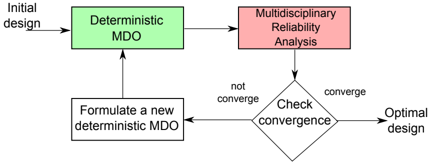


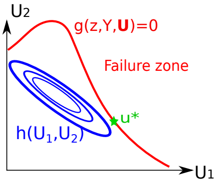
in case $U_1 \sim \mathcal{U}(0,1)$

But method strongly limited to quasi-linear problems with independent (Normal) uncertainty distributions.

Sequential Optimization and Reliability Analysis [Du et al., 2008]

Decoupling UQ and MDO :





Sequential Optimization and Reliability Analysis [Du et al., 2008]

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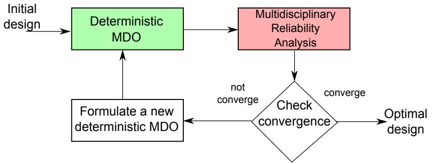

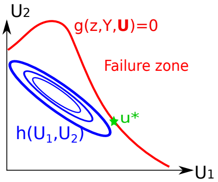

Deterministic MDO problem

$$\begin{array}{ll} \min & \mathbf{f}(\mathbf{z}, \mathbf{y}, \boldsymbol{u}^*) \\ \text{wrt} & \mathbf{z}, \mathbf{y} \\ \text{st} & \mathbf{g}(\mathbf{z}, \mathbf{y}, \boldsymbol{u}^*) \leq 0 \\ & \forall i \neq j,\ \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i, \mathbf{y}_{.i}, \boldsymbol{u}_i^*) \end{array}$$

$\mathbf{z}_{\text{opt}}$ $\longrightarrow$

$\mathbf{u}^*$ $\longleftarrow$

Search for most probable point (inverse FORM)

$$\begin{array}{ll} \max & \mathbf{h}(\mathbf{u}) \\ \text{wrt} & \mathbf{u}, \mathbf{y} \\ \text{st} & \mathbf{g}(\mathbf{z}_{\text{opt}}, \mathbf{y}, \mathbf{u}) = 0 \\ & \forall i \neq j,\ \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_{\text{opt}}, \mathbf{y}_{.i}, \mathbf{u}_i) \end{array}$$

Sequential Optimization and Reliability Analysis [Du et al., 2008]

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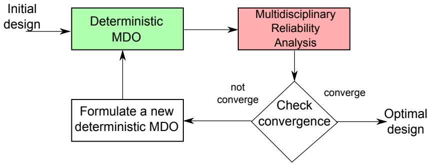

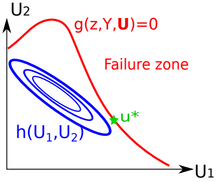

Deterministic MDO problem

min	$\mathbf{f}(\mathbf{z},\mathbf{y},\boldsymbol{u}^*)$
wrt	$\mathbf{z},\mathbf{y}$
st	$\mathbf{g}(\mathbf{z},\mathbf{y},\boldsymbol{u}^*) \leq 0$
	$\forall i \neq j,\ \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_i,\mathbf{y}_{.i},\boldsymbol{u}_i^*)$

$\mathbf{z}_{\mathbf{opt}}$ $\longrightarrow$

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max	$\mathbf{h}(\mathbf{u})$
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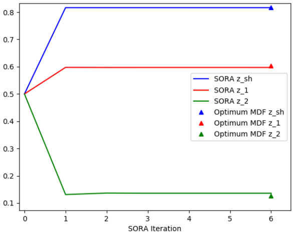
st	$\mathbf{g}(\mathbf{z}_{\mathbf{opt}},\mathbf{y},\mathbf{u}) = 0$
	$\forall i \neq j,\ \mathbf{y}_{ij} = \mathbf{c}_{ij}(\mathbf{z}_{\mathbf{opt}},\mathbf{y}_{.i},\mathbf{u}_i)$

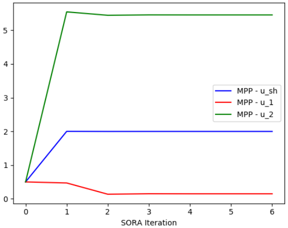
- Solving of deterministic MDO problem with uncertain variables fixed at the MPP $\boldsymbol{u}^*$,
- Search of MPP (*e.g.* FORM) : design variables are fixed at their current optimal values $\mathbf{z}_{opt}$, coupling constraints are imposed
- Generic approach that can be applied to all MDO formulations, either coupled or decoupled [Li et al., 2010, Li et al., 2014, Ahn and Kwon, 2006]

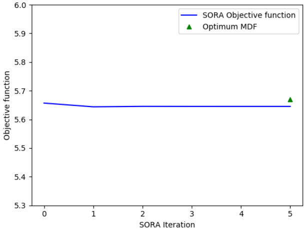
Sequential Optimization and Reliability Analysis

Comparison with MDF. Same function as before but :

- $U_{sh} \sim \mathcal{N}(2, 0.3)$
- $U_1 \sim \mathcal{U}(0, 1.)$
- $U_2 \sim \mathcal{N}(4, 0.5)$







Statistical moment matching [Liu et al., 2006]

- Coupling variables handled by the optimizer : q first statistical moments of coupling variables : $\mathbf{\Theta}[\mathbf{Y}] = \{\mathbf{M}_1[\mathbf{Y}], \ldots, \mathbf{M}_q[\mathbf{Y}]\}$ with hypothesis on their distributions.

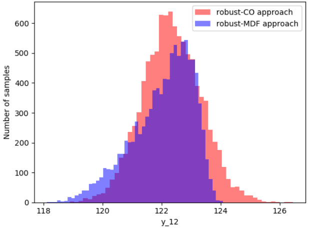
- Allow "deterministic handling" of coupling variables

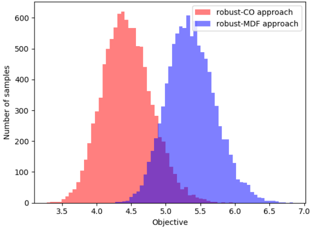
$$
\begin{array}{rc}
\text{min} & \Xi[\mathbf{f}(\mathbf{M}_1[\mathbf{Y}], \ldots, \mathbf{M}_q[\mathbf{Y}], \mathbf{z}, \mathbf{U})] \\
\text{wrt} & \mathbf{M}_1[\mathbf{Y}], \ldots, \mathbf{M}_q[\mathbf{Y}], \mathbf{z} \\
\text{st} & \mathbb{K}_\mathbf{g}[\mathbf{g}(\mathbf{M}_1[\mathbf{Y}], \ldots, \mathbf{M}_q[\mathbf{Y}], \mathbf{z}, \mathbf{U})] \leq 0 \\
& \forall j \neq i, \forall k \in \{1, \ldots, q\}\ \mathbf{M}_k[\mathbf{Y}_{ji}] = \mathbf{M}_k[\mathbf{c}_{ji}(\mathbf{M}_1[\mathbf{Y}_{.i}], \ldots, \mathbf{M}_q[\mathbf{Y}_{.i}], \mathbf{z}_j, \mathbf{U})]
\end{array}
$$

Possible improvement : Let the optimizer handle all the covariance terms of coupling variables [Ghosh et al., 2014].

Statistical moment matching, illustration

Same problem as before, use of multi-level matching moment methods, name matching moment collaborative optimization [Xiong et al., 2014].
Hypothesis of coupling variables are Normally distributed ...





... not valid in this case.

Integral form of coupling satisfaction under uncertainties

For a coupling relationship y_{ij} between disciplines i and j :

Deterministic decoupled approach :

- $y_{ij} = c_{ij}(z_i, y_{.i})$: one equality constraint

Integral form of coupling satisfaction under uncertainties

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Deterministic decoupled approach :

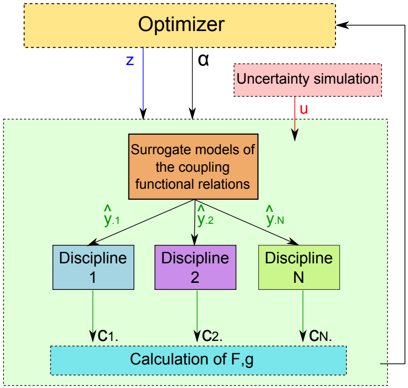
- $y_{ij} = c_{ij}(z_i, y_{.i})$: one equality constraint

Decoupled approach under uncertainties :

- $\forall \mathbf{u} \in \mathbf{\Omega},\ y_{ij}(\mathbf{u}) = c_{ij}(z_i, y_{.i}(\mathbf{u}), \mathbf{u})$: ∞ of equality constraints
- Integral form : $\int_{\Omega} \left[y_{ij}(\mathbf{u}) - c_{ij}(\mathbf{z}_i, y_{.i}(\mathbf{u}), \mathbf{u})\right]^2 h_{\mathbf{U}}(\mathbf{u}) d\mathbf{u} = 0$
- How let the optimizer handle $y_{ij}(\mathbf{u})$?
  - Use of surrogate model : Polynomial Chaos Expansion (PCE) $y_{ij}(\mathbf{u}) \rightarrow \hat{y}_{ij}(\mathbf{u}, \boldsymbol{\alpha})$
  - $\boldsymbol{\alpha}$: coefficients of PCE handled by the optimizer ($\rightarrow$ **new coupling variables**).

IDF-PCE (Individual Discipline Feasible - Polynomial Chaos Expansion) [Brevault et al., 2016, Brevault et al., 2020, Liu et al., 2022]

- Iterative training of surrogate model of coupling relationships,
- Surrogate model : Polynomial Chaos Expansion (PCE),
- Optimizer handles design variables **z** and **PCEs coefficients** α,
- Surrogate models mimic, **at the convergence of the optimization process**, coupling relationship as MDA does.



Formulation IDF-PCE

$$
\begin{aligned}
\min \quad & \Xi\left[f(\mathbf{z},\boldsymbol{\alpha},\mathbf{U})\right] && (1)\\
\text{wrt} \quad & \mathbf{z},\boldsymbol{\alpha} && (2)\\
\text{st} \quad & \boldsymbol{K}\left[\mathbf{g}(\mathbf{z},\boldsymbol{\alpha},\mathbf{U})\right] \leq 0 && (3)\\
& \forall i \neq j,\ J_{(m_{ij})} = \int_{\boldsymbol{\Omega}} \left[\mathbf{c}_{ij}\left(\mathbf{z}_i,\mathbf{u}_i,\hat{\mathbf{y}}_{.i}\left(\mathbf{u},\boldsymbol{\alpha}^{(.i)}\right)\right) - \hat{\mathbf{y}}_{ij}\left(\mathbf{u},\boldsymbol{\alpha}^{(ij)}\right)\right]^2_{(m_{ij})} h_{\mathbf{U}}(\mathbf{u})d\mathbf{u} = 0 && (4)
\end{aligned}
$$

- $\hat{\mathbf{y}}_{.i}\left(\mathbf{u},\boldsymbol{\alpha}^{(.i)}\right)$ PCEs of all coupling variables i,
- PCEs coefficients $\boldsymbol{\alpha}$ handled at system level,

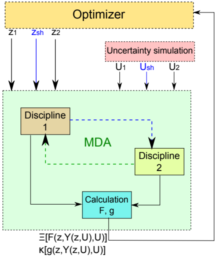
Three approaches to estimate integrals (Eq. (4)) :

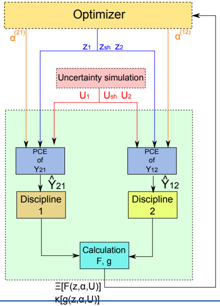
- Monte-Carlo,
- Quadratures,
- Decomposition of output coupling variables over other PCEs.

IDF - PCE, illustration

Comparison with MDF. Same function as before but :

- $U_{sh} \sim \mathcal{U}(-1, 1)$
- $U_1 \sim \mathcal{U}(0, 1)$
- $U_2 \sim \mathcal{N}(0, 1)$





Results

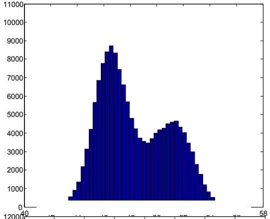
Results	**MDF-MDA (ref)**	**IDF-PCE (MC)**	**IDF-PCE (quadrature)**	**IDF-PCE (PCE)**
Objective	$\mu_F = 0.928$	$\mu_F = 0.926$	$\mu_F = 0.926$	$\mu_F = 0.914$
Design variables	$z_{sh} = 0.520$ $z_1 = 0.340$ $z_2 = 0.658$	$z_{sh} = 0.511$ $z_1 = 0.339$ $z_2 = 0.661$	$z_{sh} = 0.514$ $z_1 = 0.340$ $z_2 = 0.661$	$z_{sh} = 0.523$ $z_1 = 0.349$ $z_2 = 0.649$
Contraintes couplage	$\parallel c_{12} - y_{12} \parallel^2 \leq 0.0001$ $\parallel c_{21} - y_{21} \parallel^2 \leq 0.0001$	$J_{12} = 0.00067$ $J_{21} = 0.00057$	$J_{12} = 0.00054$ $J_{21} = 0.00074$	$\parallel \alpha^{(21)} - \tilde{\alpha}^{(21)} \parallel^2 = 0.48$ $\parallel \alpha^{(21)} - \tilde{\alpha}^{(21)} \parallel^2 = 0.3$
Nb iterations of optimization	$N_i = 2016$	$N_i = 5608$	$N_i = 5501$	$N_i = 5262$
Nb calls disciplines	$N_d = \mathbf{1512 * 10^6}$	$N_d = \mathbf{841.2 * 10^6}$	$N_d = \mathbf{3.52 * 10^6}$	$N_d = \mathbf{3.37 * 10^6}$
Speed-up factor	**1** (ref)	**1.80**	**429.55**	**448.66**

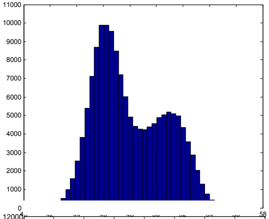
Results - distribution

MDF

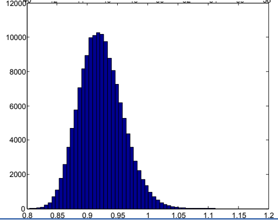
IDF - PCE

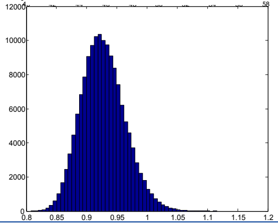
Coupling variable (Y_{21})



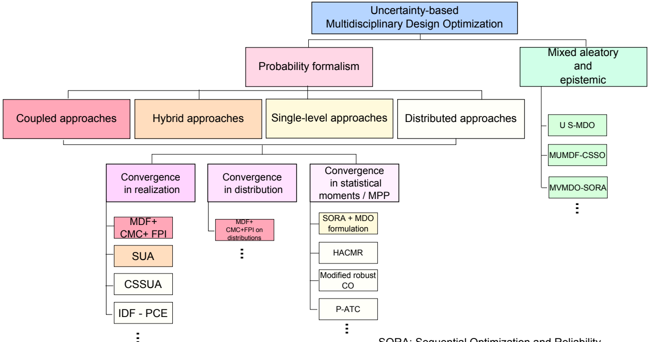


Objective function





Classification of approaches [Brevault et al., 2020]

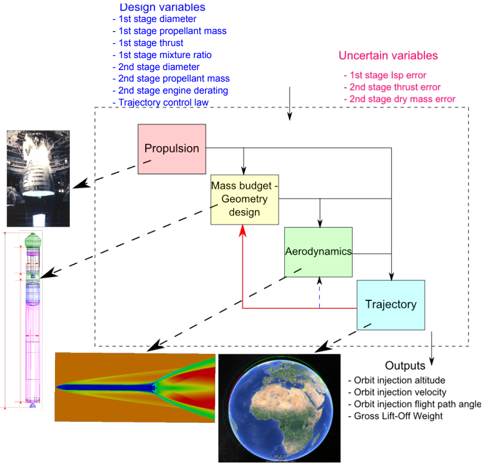


MDF: MultiDisciplinary Feasible
FPI: Fixed Point Iteration
CMC: Crude Monte Carlo
SUA: System Uncertainty Analysis
CSSUA: Concurrent SubSystem Uncertainty Analysis
IDF-PCE: Individual Discipline Feasible - Polynomial Chaos Expansion

SORA: Sequential Optimization and Reliability Assessment
HACMR: Hierarchical Approach to Collaborative Multidisciplinary Robust design
CO: Collaborative Optimization
P-ATC: Probabilistic Analytical Target Cascading
U S-MDO: Uncertainty-based Sequential-MDO
MUMDF-CSSO: Mixed Uncertainty based MDF-CSSO (Concurrent SubSpace Optimization)
MVMDO-SORA: Mixed Variable MDO - SORA

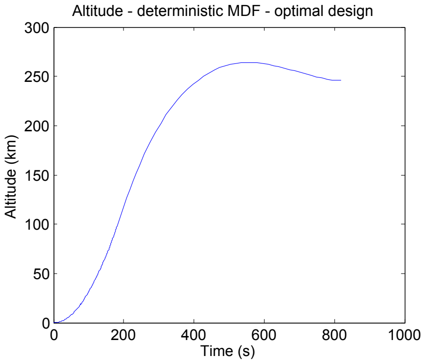
Outline

Application to space vehicle design

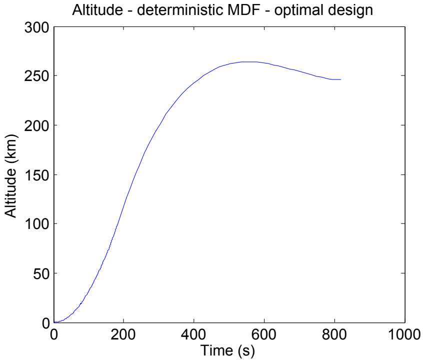


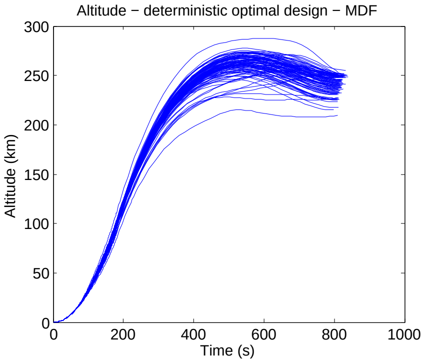
- Minimization of the expected value of the Gross Lift Off Weight (GLOW)
- Constraints : reach GTO orbit
- Number of design variables : 27
- Number of uncertain variables : 3

Deterministic solution



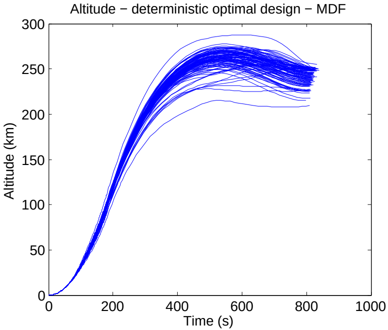
Deterministic solution





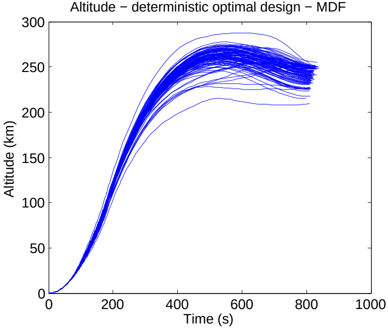
Deterministic MDO $\Rightarrow$ Non robust to uncertainties

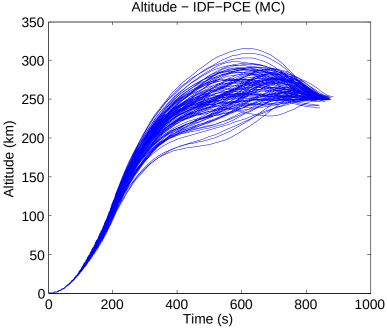
Optimal solution with handling of uncertainties



[Balesdent et al., 2016, Brevault et al., 2020]

Optimal solution with handling of uncertainties

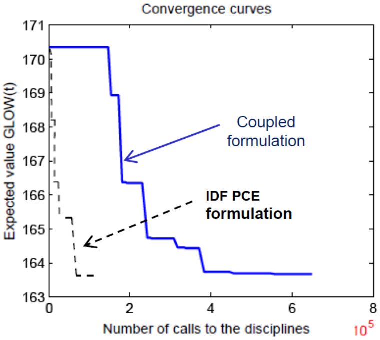


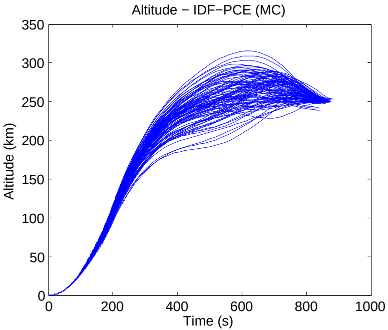


UMDO solution ⇒ improvement of robustness to uncertainties

[Balesdent et al., 2016, Brevault et al., 2020]

Optimal solution with handling of uncertainties

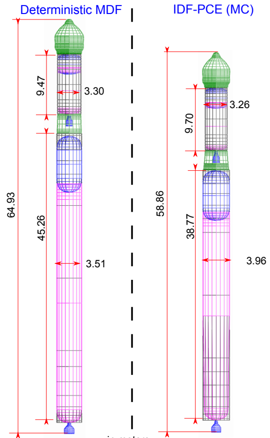




Speed up factor with respect to coupled approach : ∼ 11

[Balesdent et al., 2016, Brevault et al., 2020]

Launch vehicle geometry and 3D trajectories

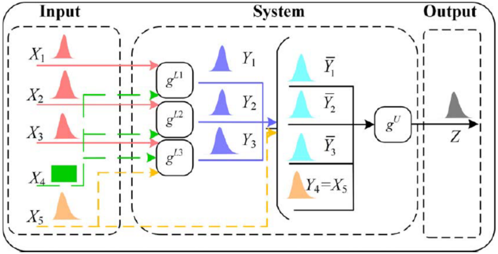


GLOW :	158.1t	163.8t

Outline

Hot topics

- Decoupled/distributed sensitivity analysis

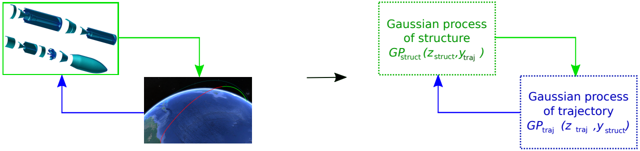


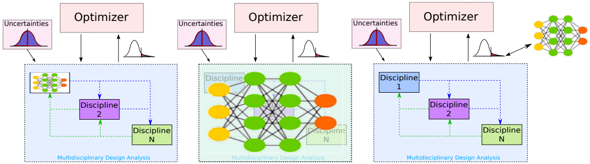
- How to use disciplinary SA results (*e.g.* Sobol indices) to provide information on global multiphysics SA ?

[Liu et al., 2021]

Hot topics

- Decoupled/distributed sensitivity analysis
- Surrogate model in the multiphysics analysis

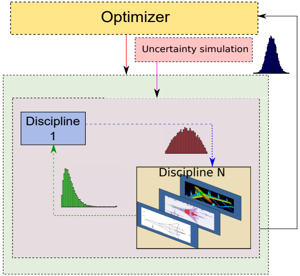




- Which surrogate model ?
- How to define adaptive strategies to refine consistently with respect to the coupling satisfaction ? [Dubreuil et al., 2020]

Hot topics

- Decoupled/distributed sensitivity analysis
- Surrogate model in the multiphysics analysis
- Multi-fidelity modeling of multiphysics system

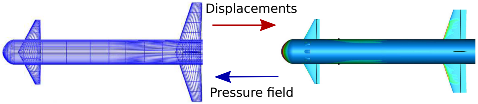


- Which physics / model to refine ? How ? At which level (*e.g.* MDA, disciplinary level ?)

[Kontogiannis and Savill, 2020, Garland et al., 2020]

Hot topics

- Decoupled/distributed sensitivity analysis
- Surrogate model in the multiphysics analysis
- Multi-fidelity modeling of multiphysics system
- Field distributed coupling variable



- How to model field coupling variables and handle them in multiphysics optimization ?

[Degroote, 2013, Berthelin et al., 2021]

Outline

Concluding remarks

- UQ for multiphysics system analysis and design is a very dynamic field of research and applied in various domains (aerospace, civil engineering, automotive, structure, *etc.*),

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Concluding remarks

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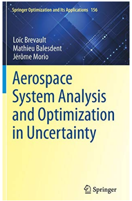
- Numerous methods allow now to perform design with uncertainties from modeling to optimization under uncertainties while accounting for multiphysics couplings,
- Several methodological breakthroughs are still locked and need further research (*e.g.* high dimensional multidisciplinary couplings handling, multi-fidelity with various input dimensions),

Concluding remarks

- UQ for multiphysics system analysis and design is a very dynamic field of research and applied in various domains (aerospace, civil engineering, automotive, structure, *etc.*),
- Numerous methods allow now to perform design with uncertainties from modeling to optimization under uncertainties while accounting for multiphysics couplings,
- Several methodological breakthroughs are still locked and need further research (*e.g.* high dimensional multidisciplinary couplings handling, multi-fidelity with various input dimensions),
- Most of current methods still need research to deal with the complexity of real world problems (*e.g.*, scale with the number of uncertain variables, of design variables, of disciplines, of field variables).

Thank you !

For more information →



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