# Statistical Parameter Calibration using Bayesian Inference applied to a Finite Element Model

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## Motivation

Considering uncertainty of structural dynamics systems to improve model predictions

| Uncertain parameters | instead of | Deterministic parameters |
| --- | --- | --- |
| $\boldsymbol{\theta} = \begin{cases} E \sim \mathrm{U}(180\ \mathrm{GPa}, 220\ \mathrm{GPa}) \\ \rho \sim \mathrm{U}(7500\ \mathrm{kg\ m^{-3}}, 8500\ \mathrm{kg\ m^{-3}}) \end{cases}$ | | $E = 210\ \mathrm{GPa}$<br>$\rho = 7850\ \mathrm{kg\ m^{-3}}$ |

## Methodology

Calibrate the uncertain parameters by application of Bayes' theorem and solving in $k$ Stages by Transitional Markov Chain Monte Carlo (TMCMC) algorithm

Posterior distribution, Likelihood function, Prior distribution

$$p_j(\boldsymbol{\theta}|\boldsymbol{D}, M) \propto p(\boldsymbol{D}|\boldsymbol{\theta}, M)^{q_j} \cdot p(\boldsymbol{\theta}|M)$$

$$j = 0,1, \ldots, k$$

$$q_j \in [0,1], q_0 = 0 < q_1 < \cdots < q_k = 1$$

[Ching, Chen: Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection and Model Averaging. In: Journal of Engineering Mechanics 133.7 (2007), S. 817–832]

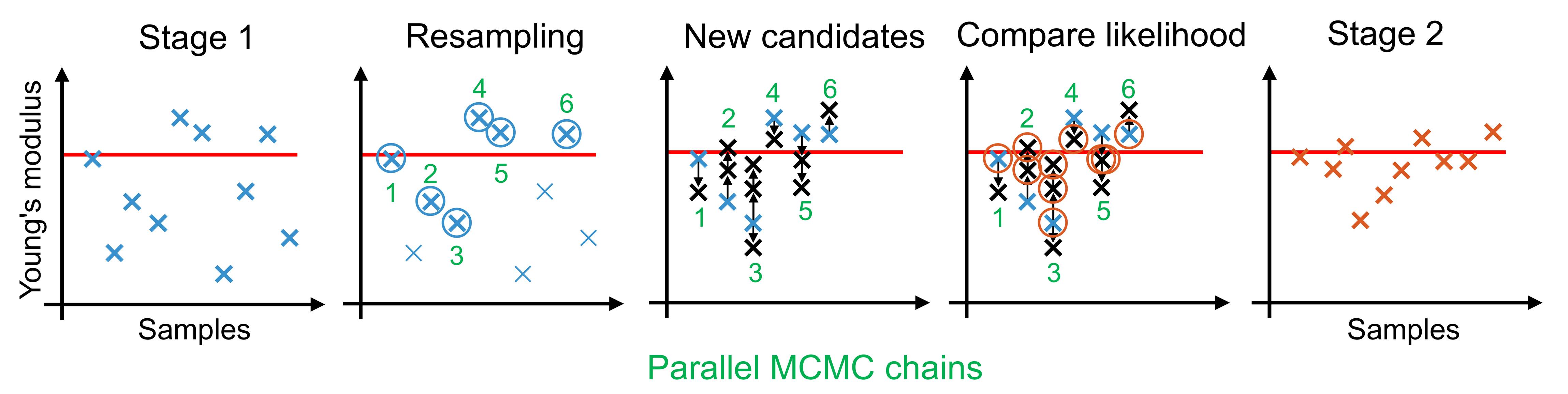
The likelihood function allows matching of experimental data $\boldsymbol{D}$ and model evaluation $M(\boldsymbol{\theta})$

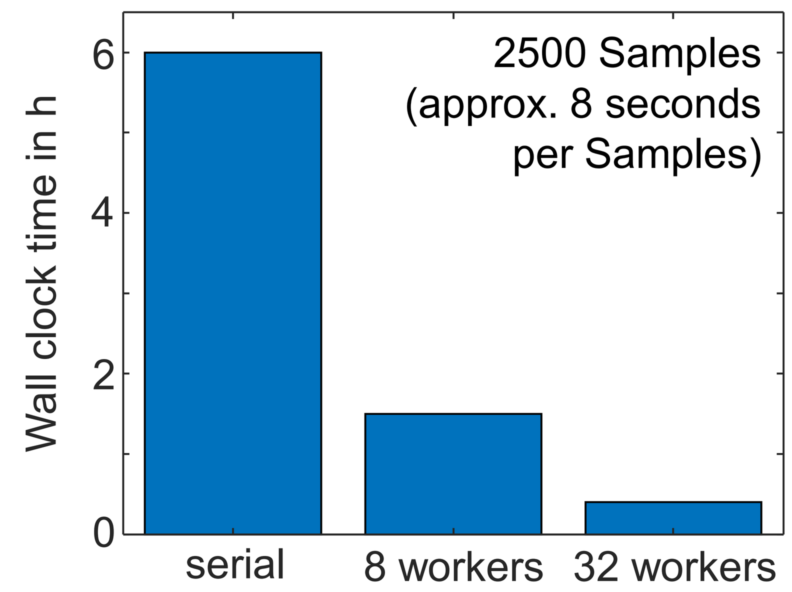
Modal Assurance Criterion

$$p(\boldsymbol{D}|\boldsymbol{\theta}, M) = \exp\left(-\frac{1}{2}\sum_{i=1}^{10}\left(\sigma_1^{-2}\left(\omega_{1,i}^{2(D)} - \omega_1^{2(M)}\right)^2 - \delta_1^{-2}\left[1 - \frac{|\boldsymbol{\varphi}_{1,i}^{(D)T}\boldsymbol{\varphi}_1^{(M)}|^2}{\boldsymbol{\varphi}_{1,i}^{(D)T}\boldsymbol{\varphi}_{1,i}^{(D)} * \boldsymbol{\varphi}_1^{(M)T}\boldsymbol{\varphi}_1^{(M)}}\right]\right)\right)$$

[Vanik, M.W., Beck, J.L. und Au, S.K. Bayesian Probabilistic Approach to Structural Health Monitoring. In: Journal of Engineering Mechanics 126.7 (2000).]

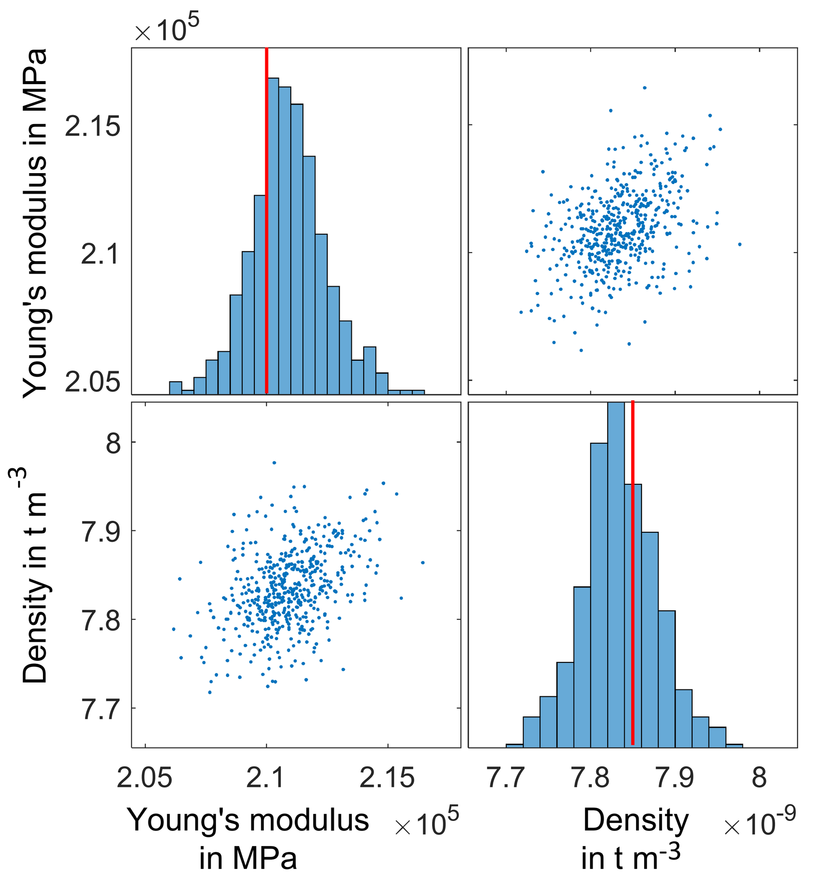
Parallelization of TMCMC algorithm to utilize high performance computer reducing calibration time



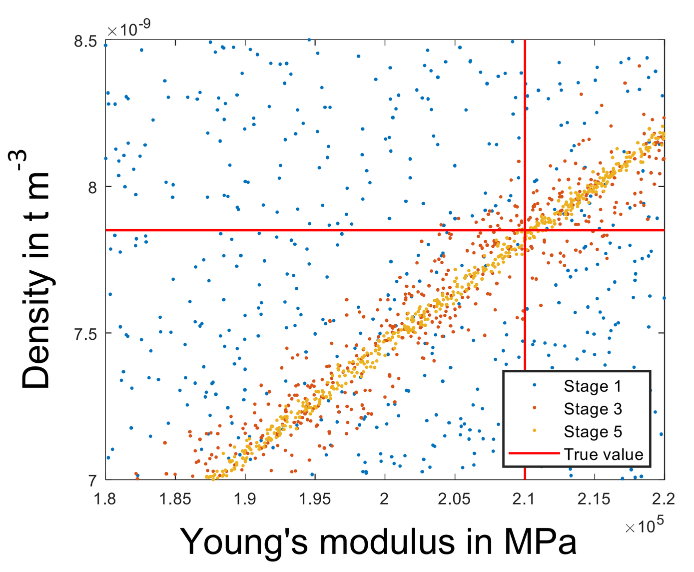

## Results

Validation by comparing results to a preset true value

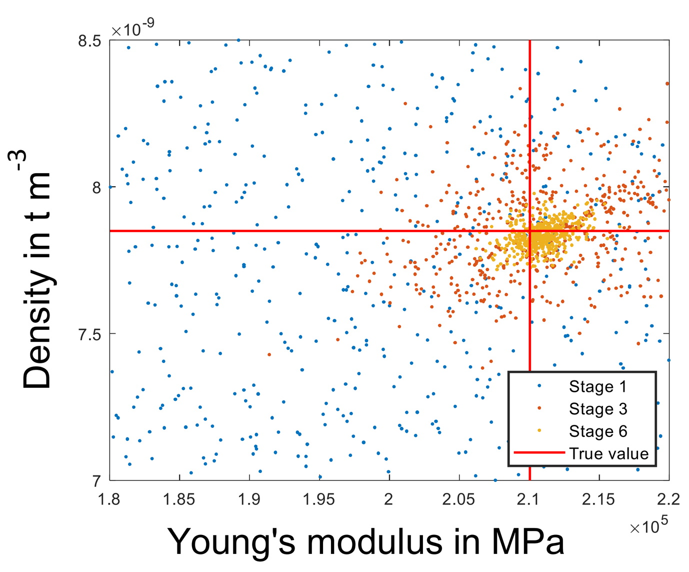



### Variation of the likelihood function

only natural angular frequency $\omega_1$




natural angular frequency $\omega_1$ and eigenvector $\boldsymbol{\varphi_1}$




frequencies $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$ and eigenvectors $\boldsymbol{\varphi_1}$,…, $\boldsymbol{\varphi_4}$