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Sensitivity to statistical estimation uncertainties and probabilistic model identification

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Context

Uncertainty propagation in engineering problems:

$$\phi : \begin{array}{ccc} \mathcal{X} \subseteq \mathbb{R}^d & \longrightarrow & \mathbb{R} \\ \boldsymbol{X} := (X_1, ..., X_d) \sim f_{\boldsymbol{X}} & \longmapsto & Y \end{array} . \quad (1)$$

Quantity of Interest (QoI):
One could be interested in assessing a given expectation of a function τ of Y by *Monte Carlo Simulation* (MCS):

$$\mathbb{E}_{f_{\boldsymbol{X}}}\left[\tau\left(\phi(\boldsymbol{X})\right)\right] = \int_{\mathcal{X}} \tau\left(\phi\left(\boldsymbol{x}\right)\right) f(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \approx \frac{1}{N_{\boldsymbol{X}}}\sum_{j=1}^{N_{\boldsymbol{X}}} \tau\left(\phi\left(\boldsymbol{X}^{(j)}\right)\right). \quad (2)$$

Industrial context:
$f_{\boldsymbol{X}}$ is estimated from a sample $\widetilde{\boldsymbol{D}}$ of limited size $N_{\boldsymbol{D}}$ [1]. The estimator (2) is subject to a bi-level uncertainty [2]:

- a first uncertainty source from the estimate $\hat{f}_{\boldsymbol{X}|\widetilde{\boldsymbol{D}}}$ of $f_{\boldsymbol{X}}$,
- a second uncertainty source from the MCS estimate.

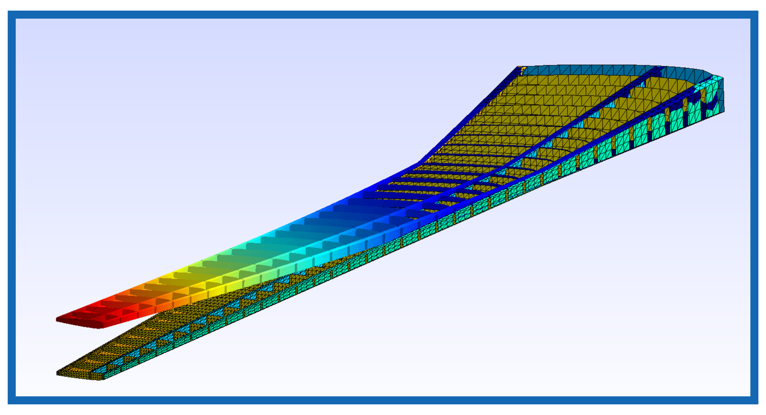
Problem

Where the investment should be made in order to reduce the variance of the estimator?

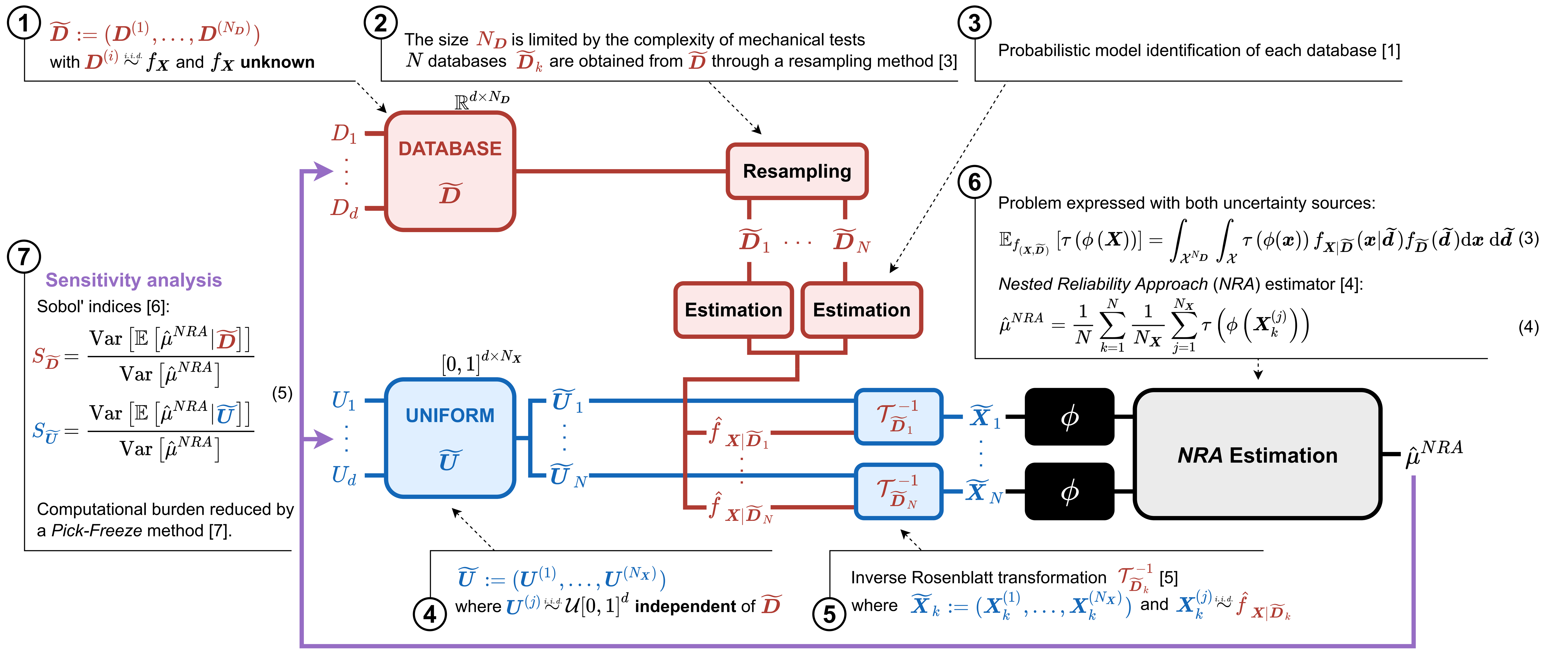
INVESTMENT

DATABASE ? $\rightarrow N_{\boldsymbol{D}} \nearrow$

SIMULATION ? $\rightarrow N_{\boldsymbol{X}} \nearrow$



The test-simulation trade-off is made based on a sensitivity analysis where the predominant indice indicates which is the leading uncertainty source on the variance of the estimator.

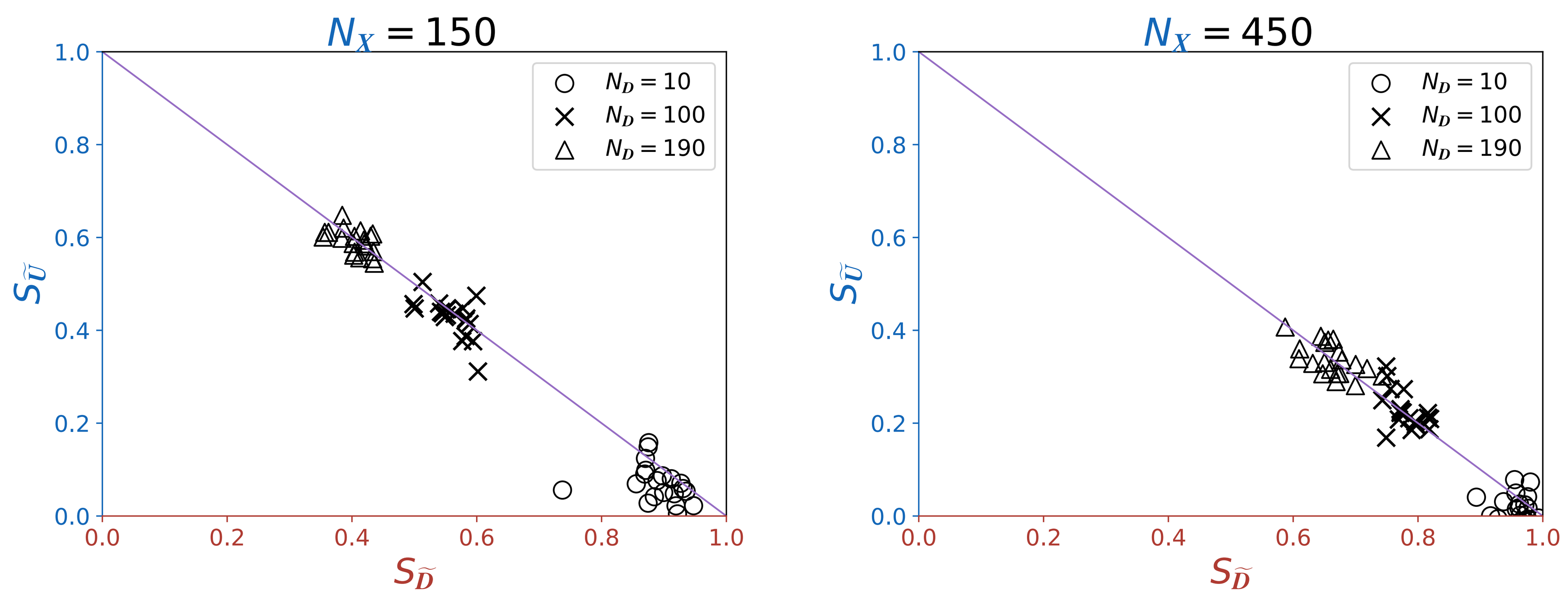


Application

Mean deflection of a cantilever beam:

$$\phi\left(F, L, E, b, h\right) = \frac{4FL^3}{Ebh^3} \quad , \quad (3)$$

where F is the transverse load applied on the free end of the beam of length L, Young's modulus E and cross-section $b\ h$ [8].



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