Uncertainty quantification and global sensitivity analysis of seismic fragility curves using kriging

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Adaptation of the UQ framework to seismic probabilistic risk assessment

- ✓Substitution of the costly mechanical computer model by a **Gaussian process surrogate**
- ✓Uncertainty propagation on a goal-oriented quantity of interest, **the seismic fragility curve**
- ✓Sensitivity analysis on parameters tainted by **epistemic** uncertainties to ensure information based design choices

Seismic fragility curve

$$\Psi(a,\mathbf{x}) = \mathbb{P}_\varepsilon(y(A,\mathbf{X}) > C | A = a, \mathbf{X} = \mathbf{x})$$

- A: Scalar variable representing the intensity of the seismic solicitation
- $\mathbf{X}$: Mechanical parameters of the structure
- C: Critical level above which the structure is considered in failure state
- $y(a,\mathbf{x}) = g(a,\mathbf{x}) + \varepsilon$: Stochastic computer model output

Kriging of computer model output

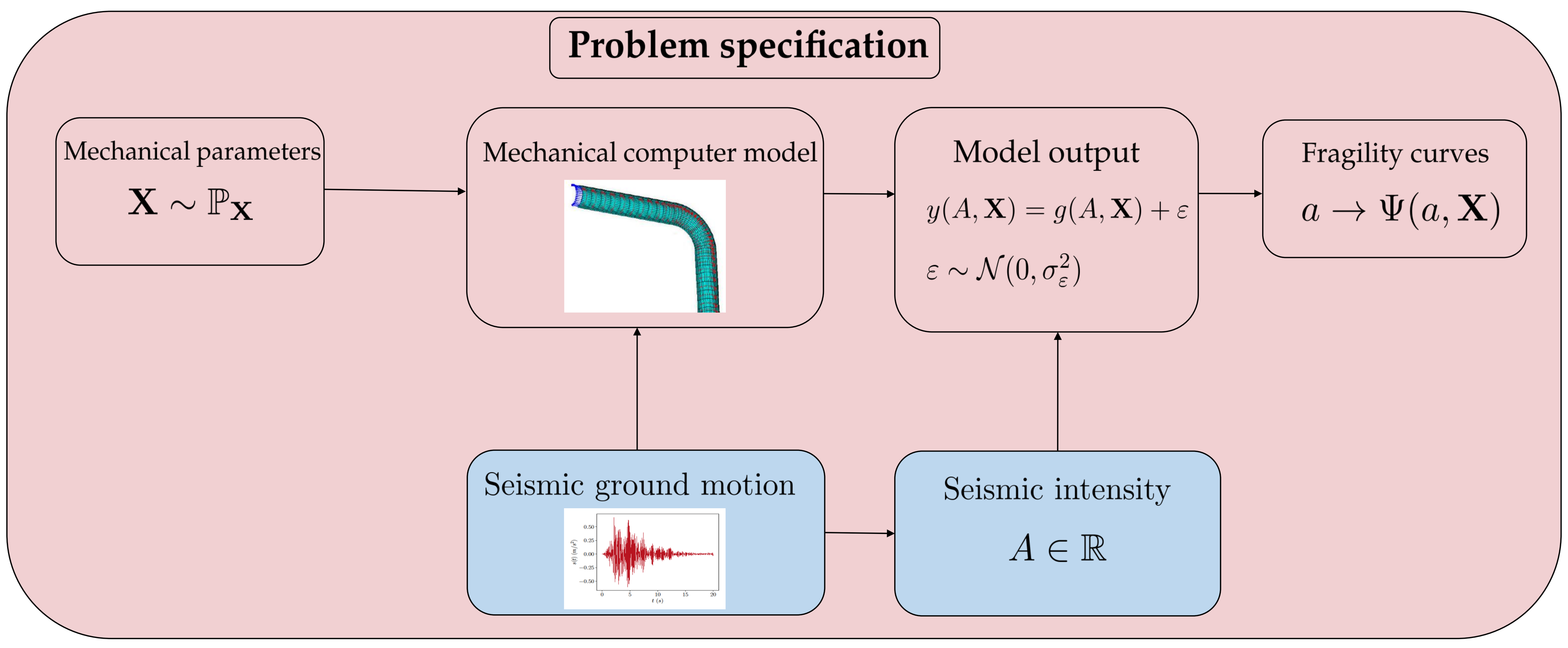
$G \sim \mathcal{GP}(0,\Sigma)$ $\quad Y(a,\mathbf{x}) = G(a,\mathbf{x}) + \varepsilon$

$\mathcal{D}_n = ((a_i,\mathbf{x}_i), y(a_i,\mathbf{x}_i))_{1\le i\le n}$

$(G|\mathcal{D}_n) \sim \mathcal{N}(\widehat{G}_n(a,\mathbf{x}), \sigma_n(a,\mathbf{x})^2)$

$G_n \sim (G|\mathcal{D}_n)$

$\widehat{\sigma}_n(a,\mathbf{x})^2 = \sigma_n(a,\mathbf{x})^2 + \sigma_\varepsilon^2$



Sensitivity analysis on fragility curves

Aggregated Sobol indices

$$\bar{\Psi} = \mathbb{E}_\mathbf{X}[\Psi(a,\mathbf{X})]$$

$$\Psi(a,X^{(i)}) = \mathbb{E}_\mathbf{X}[\Psi(a,\mathbf{X})|X^{(i)}]$$

$$S_i^{FC} = \frac{\mathbb{E}_{X^{(i)}}[\|\bar{\Psi} - \Psi(.,X^{(i)})\|^2]}{\mathbb{E}_\mathbf{X}[\|\bar{\Psi} - \Psi(.,\mathbf{X})\|^2]}$$

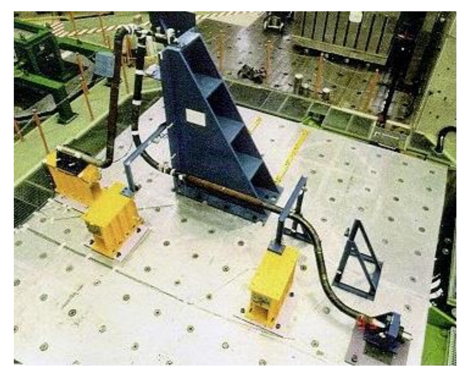
Kernel based sensitivity indices

$$\mathrm{MMD}(\mathbb{P},\mathbb{Q})^2 = \mathbb{E}[k(U,U')] + \mathbb{E}[k(V,V')] - 2\mathbb{E}[k(U,V)]$$

$$\beta_i^k = \frac{\mathbb{E}_{X^{(i)}}[\mathrm{MMD}(\mathbb{P}_\Psi,\mathbb{P}_{\Psi|X^{(i)}})^2]}{\mathbb{E}_\mathbf{X}[\mathrm{MMD}(\mathbb{P}_\Psi,\mathbb{P}_{\Psi|\mathbf{X}})^2]}$$

Application to a French pressurized reactor piping system

- Mock up of a safety water piping system of a nuclear power plant
- A nonlinear FE model is calibrated using real experiments



Variable number	*Variable*	*Mean*
1	Young modulus	$1.9236\ 10^{11}$ Pa
2	Elasticity limit	300 MPa
3	Hardening module	$4.27\ 10^8$
4	Modal damping ratio	1%
5	Rotational stiffness for the P151 guide in Y direction	$1.1\ 10^5$ Nm/rad
6	Rotational stiffness for the P29 clamped end in X direction	$1.1\ 10^5$ Nm/rad
7	Rotational stiffness for the P29 clamped end in Y direction	$3.3\ 10^5$ Nm/rad
8	Translational stiffness for the P29 clamped end in X direction	$1.0\ 10^6$ N/m
9	Translational stiffness for the P29 clamped end in Y direction	$2.0\ 10^5$ N/m
10	Translational stiffness for the P29 clamped end in Z direction	$1.0\ 10^6$ N/m

- ➢ Boundary conditions are relaxed on the clamped end of the mock-up in order to match the modal characteristics of the overall piping system
- ➢ Each variable is independent and follows a uniform distribution with a 15% relative variation (maximum entropy principle)
- ➢ After a screening step (HSIC based independence test), the variables 1,2,3,8,9 and 10 are considered influential.

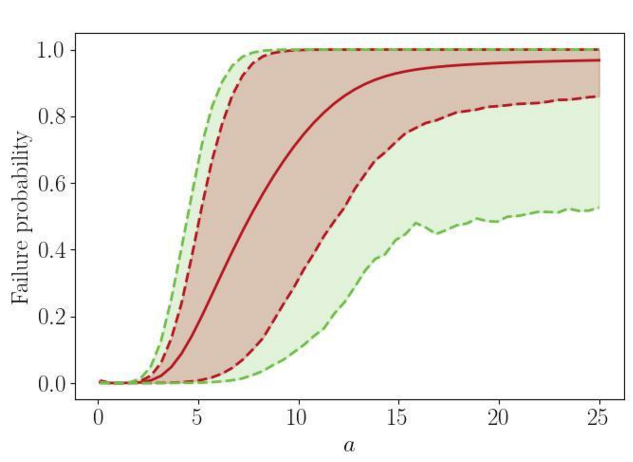
Numerical results

- ➢ Good predictivity of the kriging surrogate (Q^2 = 86 %)
- ➢ The kriging surrogate provides an uncertainty on the estimated fragility curves

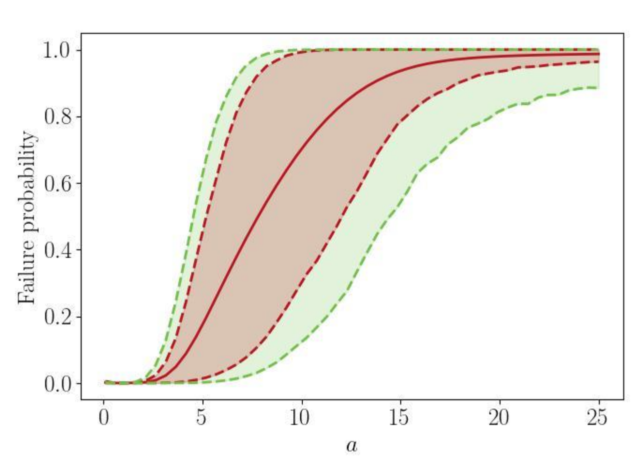
Fragility curve kriging estimator: $\Psi^{(1)}(a,\mathbf{x}) = \Phi\left(\frac{\widehat{G}_n(a,\mathbf{x}) - \log(C)}{\widehat{\sigma}_n(a,\mathbf{x})}\right)$

Fragility curve posterior distribution: $\Psi^{(2)}(a,\mathbf{x}) = \Phi\left(\frac{G_n(a,\mathbf{x}) - \log(C)}{\sigma_\varepsilon}\right)$

$$\Psi^{(1)} = \mathbb{E}_{G_n}[\Psi^{(2)}]$$

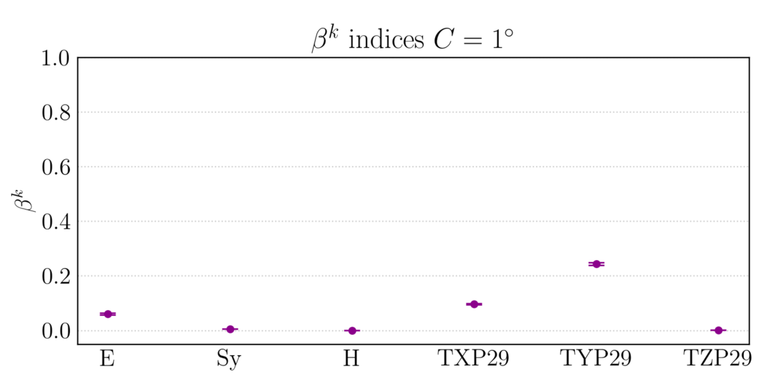


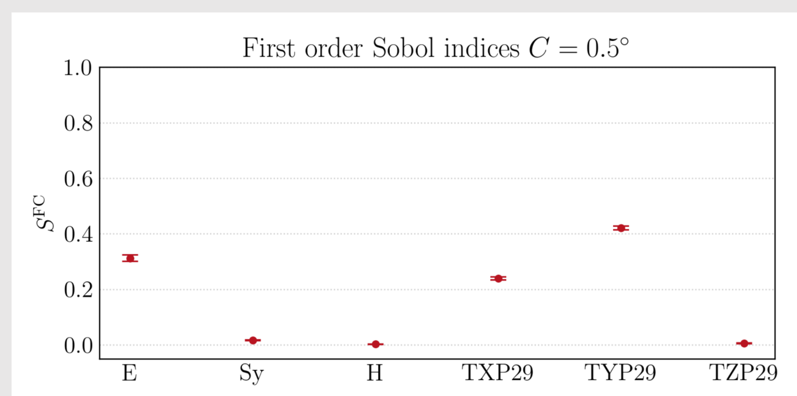
$n = 200$

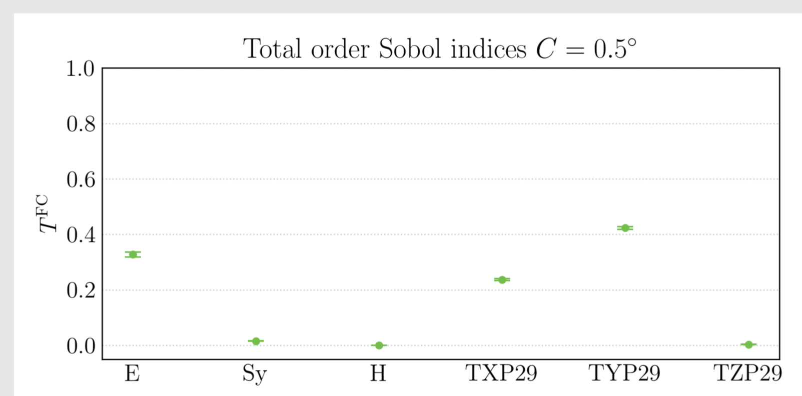


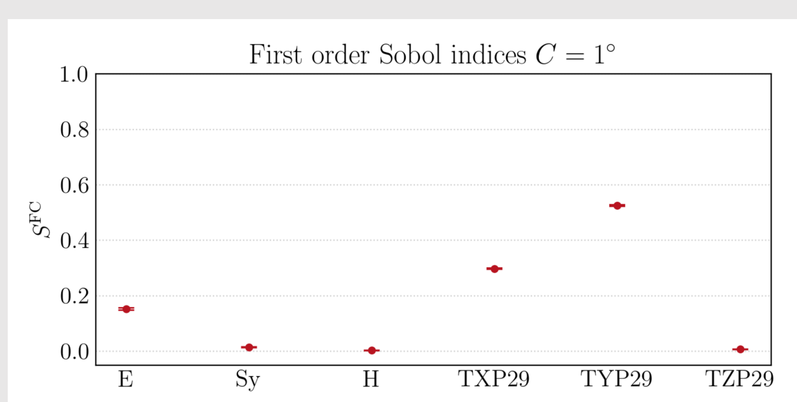
$n = 300$

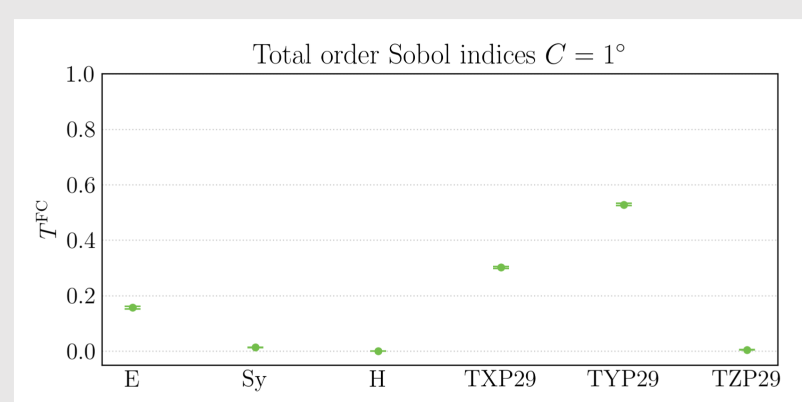
- ➢ Pick freeze estimation of aggregated Sobol first and total order indices
- ➢ **Conditional sampling** of the kriging surrogate to obtain confidence intervals on the sensitivity indices
- ➢ **Bootstrap** method to assess the pick freeze Monte Carlo uncertainty











To go further: Sequential planning of experiments

References

- John Barr and Herschel Rabitz. A Generalized Kernel Method for Global Sensitivity Analysis.*SIAM/ASA Journal on Uncertainty Quantification*, 10(1):27–54, February 2022
- Bertrand Iooss and Loïc Le Gratiet. Uncertainty and sensitivity analysis of functional risk curves based on Gaussian processes. *Reliability Engineering & System Safety*, 187:58–66, July 2019
- Loic Le Gratiet, Claire Cannamela, and Bertrand Iooss. A Bayesian Approach for Global Sensitivity Analysis of (Multifidelity) Computer Codes. *SIAM/ASA Journal on Uncertainty Quantification*, 2(1):336–363, January 2014.

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