Kriging Adaptive Learning for High Dimensional Reliability Assessment with a Variance-based Learning and Stopping criterion

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Introduction

- Probabilistic approaches received growing interest in research community and industry during last decades
- Increasing complexity of industrial numerical model lead to higher and higher computational burden
- Machine Learning (ML) techniques are proven to ease the computational burden of reliability analyses
- Real-world applications often imply a high number of input variables, making the ML-based approaches more challenging

Challenges

- Determine failure probability $P_f$
- Reference: Monte Carlo (MC)
  - Pros: no curse of dimensionality
  - Cons: requires too many simulations of (often) computationally expensive models
- Alternative: replace model with Kriging
  - Pros: optimal adaptive construction
  - Cons: curse of dimensionality
- Objectives: cost saving & accuracy
  - Capability to efficiently determine $P_f$
  - Handle high input dimensionality
  - Provide a global measure of accuracy
  - Balance accuracy/computational burden

Proposed Mathematical Framework

The global algorithm architecture is inspired from [4]. Scalable surrogate model
- KPLS [2] → extend Kriging advantages to high-dimensional problems
- Fully-adaptive algorithm
  - Adaptive learning → iteratively enrich DoE
  - Adaptive sampling → iteratively enrich MC
Variance decomposition
- $V_G$ → variance due to KPLS
- $V_X$ → variance due to MC
- $V_{G\times X}$ → covariance term (assumed negligible)
Variance-based learning criterion
- $EFF$ function [1] → select new point
- Variance-based ($Vb$) criterion → $V_G \leq V_X$
Variance-based stopping criterion
- Total variance $\rightarrow V_G + V_X \leq \bar{V} = c_0 Vf$
- Bootstrap → verify that $V_{G\times X}$ can be neglected
  - Used for sampling and overall procedure

Surrogate model [2]

Kriging
+ Good exploration features
+ Proper error structure
- Curse of dimensionality
Training/prediction based on covariance kernel definition:

$$k(x,x') = \sigma^2 \prod_{i=1}^{d} \exp(-\theta_i (x_i - x'_i)^2)$$

Partial Least Square
+ Allows dimensionality reduction
- Limited accuracy
Projection of input variables:

$$l^{(b)} = X W^s \quad y \approx c t^{(b)}$$

Kriging Partial Least Square
+ Good exploration features
+ Proper error structure
+ Scalable for high dimensional inputs
Modified covariance kernel definition:

$$k(x,x') = \sigma^2 \prod_{l=1}^{d} \prod_{i=1}^{d} \exp(-\theta_i (w^{(l)}_{ij} x_i - w^{(l)}_{ij} x'_i)^2)$$

References


Conclusion

We achieved the main objective of conceiving a ML-based approach for reliability applications:
- scalable, thus able to extend Kriging exploration features to high-dimensional problems
- efficiently updating the training set with the most relevant inputs
- reducing computational efforts with respect to other referenced methods
- able to distinguish the different sources of error, in terms of variance, of the $P_f$ estimation
- providing a global measure of variability of the failure probability estimate
- guaranteeing a good overall accuracy by balancing the sources of uncertainty.

Numerical Results

Comparison with $U$ (from AK-MCS [3]) and $EFF$ approach [1].

Evolution of $P_f$ as function of DoE updates

Variance decomposition for $EFF$

Variance decomposition for $Vb$