MASCOT-NUM 2022
Clermont Ferrand, France
7-9 June 2022

Physics Informed Neural Networks for inverse uncertainty quantification of material elastic properties on a simple 2D beam example

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PhD expected duration: Nov. 2021 - Sep. 2024

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Abstract:

In the field of uncertainty quantification, the need to perform many simulations often becomes prohibitively time consuming, especially for complex models. Surrogate modelling is a widely used technique to tackle this issue. In this approach, the high fidelity model (HFM) is replaced by a simplified one that relies on assumption on the form of the output, and requires much less computing resources. Popular models include polynomial chaos expansion, Kriging and support vector machines. An overview can be found in Sudret et al. [6]. Nevertheless, the simplification assumptions can be hard to justify and are in any case limited when used on real case applications, leading to inevitable interpolations errors. Quantifying exactly these errors and their impacts on the uncertainty quantification results remains a delicate issue that often requires case-by-case study.

Recently, a new framework called Physics-Informed Neural Network (PINNs) has emerged as a new branch of the very active deep learning research field [2] and appears to offer great potential to be used as surrogate model for uncertainty quantification. Introduced in 2019 by Raissi et al. [5], this framework allows neural network to be more robust by having them complying to some prior physics laws described by partial differential equation (PDE). The PINN is indeed trained to minimize both prediction error and a residual error computed by injecting the neural network in the differential equation using automatic differentiation. As a result, solving partial differential equations with PINNs combine the advantages of data-driven and physics-based approaches, being both resource-efficient and faithful to physics. They could therefore become good alternatives to the more resource-intensive finite element method.

The potential of PINNs for uncertainty quantification is apparent in the several publications published on this precise subject since the introduction of the framework [7, 4], including papers on combinations with other techniques like polynomial chaos expansion in Zhang et al. [8]. A concrete application of PINNs to model the physical response of a continuous medium has been achieved in Haghighat et al. [1].

Despite their ease of implementation, they are still limited by the difficulties associated with their training. The convergence of their optimization is notoriously sensitive to minor changes in the system and in a way that remains unpredictable so far. The convergence depends on the classic settings neural-network settings (choice of architecture, loss function, otpimizer, tec.) but also on hyper-parameters specific to PINNs, notably:

- the choice of points to calculate the prediction error and the error due to the PDE
- how to impose the boundary and initial conditions of the problem
- how to combine the different losses

A lot of work can be found in the recent literature on exploring this hyper-parameters and better understanding the optimization process. For instance, a curriculum learning approach is proposed in [3] but many grey areas remain to be clarified.

In this preliminary work we have considered the simple case of a 2D elastic beam under different boundary conditions. We will present how a physical approach can improve the challenging optimization step. The obtained model is then used to quantify the spatial uncertainty of the elastic properties of the material using a Monte Carlo method.

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Short biography – I am a french engineer doing a PhD at KU Leuven within the Greydient ITN project. (This project has received funding from the European Union's Horizon 2020 Research and Innovation programme under grant agreement n°955393) My goal is to develop grey-box inverse identification algorithms to characterize the properties of materials. I am working in partnership with the company MatchID which develops a Digital Image Correlation (DIC) technology that allows to measure entire deformation fields.