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Statistical Parameter Calibration using Bayesian Inference applied to a Finite Element Model

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Abstract:

Models are used to describe the behavior of a technical system under specific aspects. When parameters of the model are defined deterministically, the prediction accuracy of the models deteriorates, since the uncertainty of the model parameters is ignored. In respect of an increase in virtualization and a decrease in experimental validation, quantifying model prediction accuracy is becoming more important. The quantification of model parameters uncertainty can be achieved by a statistical parameter calibration using Bayes' theorem, whose numerical solution is possible via the implementation of Markov-Chain-Monte Carlo algorithms [7]. For the application of this method, the model must be evaluated several thousand times while changing the uncertain parameters between simulations. Due to the increasing complexity of models, this can lead to unbearable high computing times. This problem can be addressed by parallelizing the model simulations using efficient algorithms such as the Transitional Markov Chain Monte Carlo Algorithm [3, 2] or integrating surrogate models like a kriging model [1].

Finite Element models are commonly used models in product engineering. They contain a large number of parameters defining the models' geometry, material, and boundary conditions. It is usually not possible to assess the true value of these parameters, because of their inherent variability and randomness. For example, dimensions are influenced by production tolerance and material parameters might differ between batches. Since these parameters are unknown concerning their statistical distribution, it is a common approach to describe them by the mean or extreme value which is causing uncertainty. The uncertainties within the model parameters result in uncertainties in the model output. To achieve robust response predictions through a FE analysis a rational approach has to include these unavoidable uncertainties [6]. Uncertainty of model parameters belongs to the category of data uncertainty. But it is noted, that when applying FE models also other types of uncertainty occur, such as through the linear modeling of nonlinear interactions or ignoring of elasticity at bearings or joints. These can be assigned under the term of model uncertainty [5].

To statistically describe model parameters by a probability distribution, a large amount of data is necessary. However, available data is usually scarce, so the information is limited to mean values or upper and lower bounds. Under these circumstances, it is reasonable to start selecting a convenient distribution like a uniform or normal distribution to describe the uncertain model parameters regarding a prior distribution. In general, it is recommended to assume all model parameters to be uncertain parameters, unless their influence has been previously investigated by a sensitivity analysis. The computational effort for the calibration is independent of the number of variables introduced while considering all uncertainties ensures a robust prediction without distorting the result by prejudice. The implemented method for parameter calibration allows updating the existing information about the model parameters when new information becomes available. As a source of new information results from experiments or high fidelity model simulations are conceivable. The fusion of different sources of information is performed by application of Bayes' theorem. The result of the calibration is an update of the prior distribution into the posterior distribution. This is performed under consideration of a likelihood function, which describes the degree of agreement between the two sources of information. The function is aimed to be maximized during the calibration. It is determined during calibration by simulating a reference quantity for different

parameter values. Eigenfrequencies and eigenvalues as a result of modal analysis are used as reference quantities.

To solve the Bayes' theorem numerical methods are applied to avoid high-dimensional integrations. The Markov Chain Monte Carlo Algorithm is based on Markov Chains on the model parameters space whose steady-state distribution is the distribution of interest. The solution consists of samples that are in the same distribution as samples that would have been drawn directly from the posterior. Similar to the samples resulting from a classical Monte Carlo simulation, also MCMC samples can be used for further inferences. Estimates and uncertainties are approximated by sample averages or standard deviations. The quality of the approximation depends on the size of the sample and the degree of dependence in the sample [4]. The TMCMC Algorithm is applied preferably over the Metropolis-Hastings Algorithm. Its advantages are its applicability to multimodal, very peaked, and flat distributions, and its efficiency towards higher-dimensional parameter space. Furthermore, the TMCMC Algorithmen is parallelized, to simulate and assess the likelihood of samples in parallel on a high-performance computer to significantly reduce the calibration time. Moreover, autonomous communication between the running Algorithm and the FE Solver has been established.

References

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Short biography – Ph.D. Student at the Technical University of Darmstadt working on Material qualification in Fuel Cell Cooling Systems (starting June 2022); Master's degree in Mechanical Engineering on the topic of Statistical Parameter Calibration using Bayesian Inference (April 2022); Bachelor's degree in Mechanical Engineering on the topic of Sensor Calibration of Magnetic Bearings (August 2017)