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A scalable benchmark for multidisciplinary design optimization under uncertainty

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Abstract:

Multidisciplinary Design Optimization (MDO) is a set of methods for the design by optimization of complex systems composed of several coupled subsystems (called *disciplines*). MDO turns out to be a much more efficient approach for understanding systems than sequential methods [1]. Indeed, the problem is that some disciplines may have conflicting goals. Therefore, a sequential design often results in a suboptimal if not unfeasible system. This explains the growing popularity of MDO methods for industrial and engineering applications, these approaches being already used in a large number of industrial fields such as aviation, satellites, racing cars, rockets [2] ...

The introduction of uncertain parameters in MDO problems is an active research topic in this field [3]. *MDO under uncertainty* (UMDO) considerably increases the complexity of this kind of problems since probabilistic assessment requires repeated runs of the deterministic analysis. For example, Monte Carlo sampling turns out to be unaffordable in the case of expensive codes. This pitfall can be overcome by developing specific algorithms such as [...] [4]. However, these new methods are often tested on industrial use-cases and their performances are difficult to assess. More generally, the engineering context in which MDO themes are addressed leads to the fact that practical approaches and empirical examples prevail over in-depth mathematical studies. Thus, our aim is to propose both a mathematical formalism and a scalable benchmark in order to evaluate the UMDO techniques.

We first present a mathematical formulation of an MDO problem whose UMDO form is only the natural extension. Let us consider an optimization problem that seeks to minimize a cost function $f : \mathcal{X} \to \mathcal{F} \subset \mathbb{R}$ subject to inequality constraints defined by a function g. The input variable $x \in \mathcal{X}$ is called *control variable* or *design variable*. When this design variable is decomposed into $N+1$ design variables $x_0, x_1 \dots, x_N$ (each $x_i \in \mathcal{X}_i$, $i = 0, \dots, N$) according to N underlying functions $\phi_1, \dots, \phi_N$ called *disciplines*, the optimization problem is then called a MDO problem. The output variable of ϕ_i is denoted y_i ($y_i \in \mathcal{Y}_i$) and is constrained to be an input of the other disciplines. For this reason y_i is called *coupling variable*. Thus, ϕ_i depends on the vector formed by all the coupling variables but i, denoted $y_{-i} := (y_j)_{\substack{1 \leq j \leq N \\ j \neq i}} \in \mathcal{Y}_{-i} := \mathcal{Y}_{\substack{1 \leq j \leq N \\ j \neq i}}$. It also depends on the design variables x_0 and x_i where x_0 is shared by the N disciplines. Hence, the coupling equations are written: $y_i = \phi_i(x_0, x_i, y_{-i}), \quad i = 1, \dots, N$. Similarly, the uncertain variable U can be split into random variables $U_0, U_1, \dots, U_N$. We restricted ourselves to the case where the coupling equations are systematically solved at each iteration of the global optimizer, which is called the *Multi Discipline Feasible* formulation (MDF) [5]. Thereby, the coupling variables are function of the design variables $y = c(x)$. Let $\mathbb{K}_f$ and $\mathbb{K}_g$ be statistics, typically $\mathbb{K}_f[\cdot] = \mathbb{E}[\cdot]$, $\mathbb{K}_g[\cdot] = \mathbb{P}[\cdot \geq 0] - \epsilon$ where ϵ is a fixed threshold. Finally, the uncertain version of the MDF problem (UMDF) is written :

$$
\begin{aligned}
\underset{x \in \mathcal{X}}{\text{minimize}} \quad & \mathbb{K}_f[f(x_0, U_0, c(x, U))] \\
\text{subject to} \quad & \mathbb{K}_{g_0}[g_0(x_0, U_0, c(x, U))] \leq 0, \\
& \mathbb{K}_{g_i}[g_i(x_0, x_i, U_0, U_i, c_i(x, U))] \leq 0, \quad i = 1, \dots, N
\end{aligned}
\tag{1}
$$

Second, we present an analytical scalable benchmark that allows to assess the performances of UMDO methods. To our knowledge, there is no MDO benchmark in the literature on which the analytical solution is known, regardless of the number of design variables or the number of disciplines. Starting from a use-case introduced by Tedford and Martins [6], we reduce it from Problem (1) to a convex *Quadratic Programing* (QP) problem :

$$\begin{aligned} \underset{x \in \mathcal{X}}{\text{minimize}} \quad & \mathbb{K}_f \left[\frac{1}{2} x^T Q x + c^T x + d + U^T D^T D U \right] \\ \text{subject to} \quad & \mathbb{K}_g \left[Ax - b - DU \right] \leq 0 \end{aligned} \tag{2}$$

on which it is possible to find deterministic exact solutions depending of the choices of $\mathbb{K}_f$ and $\mathbb{K}_g$ [7]. By using the framework developed in the multidisciplinary optimization library GEMSEO [8], we obtain a useful benchmarking environment which will allow to analyze a large class of more complex UMDO problems. The advantages of this benchmark lie in its scalability, its fast evaluation time, and a high-level standard interface. This approach thus allows the systematic study of UMDO problems, and opens the way to evaluate new ideas concerning the use of bi-level MDO formulations in an uncertain framework, a very promising avenue in this field of research.

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Short biography – I graduated from ISAE-SUPAERO, and hold a master's degree in mathematics and statistics from Paul Sabatier University. My PhD mainly deals with multidisciplinary optimization (MDO) and uncertainty quantification. More precisely, a main research line is the development of a bi-level MDO formulation under uncertainty for the design of complex systems. The thesis is funded by IRT Saint Exupéry, a research institute which works with industrial partners on the implementation of scientific methods in an industrial context.