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Improved cross entropy method with Bernoulli mixture model

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Abstract:

Infrastructure networks, such as power grids and water supply systems are essential for societies. Therefore, it is essential to quantify how infrastructure networks perform under hazards, e.g., earthquakes or storms. The quantification of the reliability, or conversely, the probability of failure of such systems is crucial for managing their reliability. This is the main purpose of network system reliability assessment. Over the years, multiple methods have been proposed for solving the network reliability problems, among which sampling based methods such as Monte Carlo simulation and its variants feature prominently. For rare event simulation, crude Monte Carlo method becomes impractical because its computational cost for a fixed target accuracy is inversely proportional to the magnitude of the rare-event probability. A prominent alternative is importance sampling, which generates samples from an alternative distribution, the so called importance sampling distribution, and estimates the rare event probability by a weighted sample estimate. The performance of importance sampling strongly depends on the choice of the importance sampling distribution. The theoretically optimal importance sampling distribution is only known proportionally to an unknown constant, and hence, cannot be employed directly. The cross entropy method is an adaptive importance sampling method that determines a near-optimal importance sampling distribution, through minimizing the Kullback-Leibler divergence between the theoretically optimal importance sampling distribution and a predefined parametric family of distributions [1]. The performance of the classical cross entropy method can be further enhanced by introducing a smooth transition from the nominal distribution to the theoretically optimal importance distribution, which forms the basic idea of the improved cross entropy method [2].

Recently, the authors developed a method for network reliability assessment based on the improved cross entropy method that employs the independent Bernoulli model for estimating the reliability of networks with binary component states [3]. This approach has the limitation that the independent Bernoulli distribution is not able to model dependence between components. Yet, the components of the theoretically optimal importance sampling distribution of complex network systems may be strongly dependent. To address this issue, this work proposes to incorporate the Bernoulli mixture model as a parametric family within the improved cross entropy method for network reliability.

For updating the parameters of the Bernoulli mixture model in the improved cross entropy method, we propose a modified version of the expectation-maximization algorithm that works with weighted samples. The solution to the expectation-maximization algorithm highly depends on the initial assignment of samples to the clusters in the mixture [4]. For finding a good starting point of the initial assignment, we launch short pilot runs of the expectation-maximization algorithm with randomly assigned samples and choose the assignment resulting in the maximum observed log-likelihood as the starting point of the expectation-maximization algorithm. To estimate the number of clusters in the mixture, a model selection technique is employed, whereby the number of clusters leading to the smallest Bayesian information criterion is selected.

We investigate the efficiency and accuracy of the proposed approach on a set of numerical examples. The main conclusions of the numerical study are:

1. The improved cross entropy method with an independent Bernoulli model or Bernoulli mixture

model fails to work well for network reliability problems with a large number of components due to the degeneracy of the importance sampling weights.

2. For problems with moderate number of components, the improved cross entropy method using Bernoulli mixture outperforms the one using the independent Bernoulli model. The improvement is marginal when the system components are not strongly dependent conditional on the system failure.

References

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Short biography – I am a Ph.D. student from Engineering Risk Analysis Group at Technical University of Munich. Prior to this, I received my M.Sc and B.E in civil engineering from Tongji University in China. My research interests lie in advanced sampling based methods and network reliability assessment. My Ph.D. is funded by the China Scholarship Council.