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Kriging Adaptive Learning for High Dimensional Reliability Assessment with a Variance-based Learning and Stopping criterion

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Abstract:

In recent developments in probabilistic design, reliability approaches assisted by adaptive learning have been receiving growing interest.

The reference reliability method, Monte Carlo (MC), allows to estimate the failure probability P_f of a complex system and provides a statistical characterization of the estimate itself [1]. On the downsides, the number of evaluations of a given model to achieve a given coefficient of variation on the P_f estimate is roughly proportional to $1/P_f$, which is impractical for complex, numerically expensive models. Two main strategies are generally adopted to reduce the computational burden: (1) modifying the sampling strategy to reduce the variance of P_f estimation; (2) replacing the original model with a surrogate model. The second strategy constitutes the main focus of the present work.

However, constructing surrogates for high dimensional problems can be challenging. In the present work we introduce a novel reliability approach based on Kriging- Partial Least Square (KPLS) [2] which extends the good exploration features of Kriging, due to its inherent uncertainty structure, to problems with high input dimensionality.

As typical in adaptive learning approaches, the database of the evaluations of the full model (i.e. the training set of the metamodel) is enriched iteratively to progressively improve the approximation of the limit state function around the limit state surface and predict the outcomes on a large MC population. Four key features fully define our proposed algorithm: the learning criterion, to select the best-suited point to enrich the training set; the learning stopping criterion, to decide whether a sufficient accuracy of the surrogate model is reached; the definition of the MC population samples used as prediction points to estimate P_f; the overall procedure convergence criterion.

The choice of the learning function which should characterize the usefulness of candidate points constitutes the first fundamental component of the algorithm itself. In our approach, we adopt the Expected Feasibility Function (EFF) [3], which was originally introduced for Kriging, and is now extended to KPLS. This is shown to make it possible to progressively and efficiently reduce the uncertainty of the P_f estimate, by concentrating simulation efforts in unexplored or promising regions around the limit state surface.

A main novelty of our proposed approach is given by the learning phase stopping criterion, inspired from [4] and based on the variance decomposition between different uncertainty sources, namely the variance due to the surrogate model (V_G) and the one due to MC population (V_X). The former, V_G, is defined as the variance of all P_f estimations provided by different trajectories of the KPLS model. The latter, V_X is computed as the variance of the P_f estimate via the mean trajectory of KPLS on the given MC population. The total variance will be given by the sum of V_G and V_X, as we show that the covariance term is usually negligible.

Moreover, as the targeted P_f is not a priori known, the size of MC population needed to reach given accuracy requirements cannot be fixed at the beginning. Therefore, the presented algorithm alternates between limit state learning and MC population enrichment phases, which is another main novelty of our approach.

Finally, the overall procedure is typically stopped when the total variance is lower than a prefixed target, corresponding for example to a coefficient of variation of P_f estimate equal to 5%.

The objective of introducing a variance-based learning and stopping criterion is to avoid an excessive unnecessary accuracy of the surrogate model approximation and instead balance the two sources of error. This can substitute the classical criteria based on the comparison of extreme values of the learning functions [5, 3] (or differences between P_f predictions in successive steps [6]) with arbitrary targets. Two variants of such criteria are presented: in the first, called Local Variance-based (LVb), each learning phase is stopped when most of variance at a given iteration is due to MC population, thus when $V_G < V_X$; in the second variant, named Global Variance-based (GVb) the term V_G is compared with half of the total variance target, to ensure a more regular convergence from the earliest stages.

A 53-dimensional numerical example related to structural reliability is analyzed to highlight performances in terms of precision and number of full simulations required to determine failure probability. Compared to existing surrogate-model assisted reliability approaches, such as [5, 3, 6], a reduction by 50 % of the number of full model evaluations was found. The two variants, LVb and GVb presented no differences in terms of overall computational burden, but the convergence of P_f estimation was more regular with the adoption of the latter.

References

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Short biography – Provided with an aeronautical engineering background, G. Capasso worked on Topology Optimization prior to starting his PhD. Two years ago, he started a PhD project, entitled "Towards semi-probabilistic approaches for built-in stresses" resulting from the collaboration between Airbus Operations SAS and Université Paul Sabatier. The presented approach is part of a statistical framework to justify and demonstrate aircraft static strength.