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Universal Inversion: Extending Universal Kriging to Include Trends in Bayesian Inverse Problems

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Abstract:

In natural sciences and engineering, one is often faced with the problem of reconstructing some unknown function u_0 from indirect data that has been generated by a known physical process. Here indirect means that we do not have access to the actual value of the function at some selected points, but only to (for example) integrals, or other linear forms of the function. Such problems are broadly known as *inverse problems*.

Inverse problems can be solved in a Bayesian way by putting a prior on the unknown u_0 (usually gaussian process priors are used) and then using the conditional distribution to approximate the unknown function. There is a rich litterature dedicated to such approaches [3].

In this work, we extend the usual Bayesian inversion framework to include (partially) known trends. These trends are modelled as linear combinations of basis functions, with a multivariate Gaussian prior on the trend coefficients. In essence, this is an extension of the usual universal kriging approach to inverse problems.

In order for universal inversion to be applicable real-world inverse problems, in particular 3 dimensional ones, we leverage modern distributed computing frameworks that allow large matrices to be stored in a distributed fashion on a computing cluster.

We demonstrate our universal inversion techniques on a large-scale gravimetric inverse problem based on data collected on Stromboli island [2], exploring how various basis functions perform.

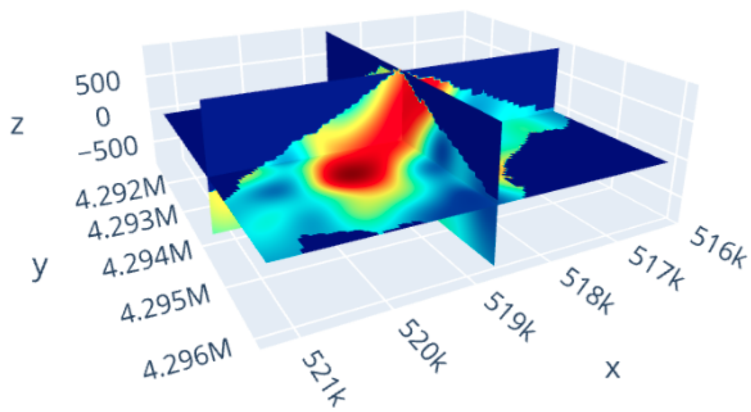

Figure 1: Stromboli volcano: Estimated underground density (posterior mean), cylindrical + planar trend.

Furthermore, we extend results from [1] to provide fast k-fold cross validation formulae for universal inversion an explore how these can be used for model selection.

Our whole Bayesian inversion machinery is distributed as an open source Python package [4] which can be used on any linear inverse problem, the user only having to provide the forward operator and the geometry of the inversion grid, while being able to choose from pre-included trend functions.

This presentation is based on joint work with David Ginsbourger and Niklas Linde.

References

[1] David Ginsbourger and Cedric Schärer. Fast calculation of gaussian process multiple-fold cross-validation residuals and their covariances. `https://arxiv.org/abs/2101.03108`.

[2] Niklas Linde, Ludovic Baron, Tullio Ricci, Anthony Finizola, André Revil, Filippo Muccini, Luca Cocchi, and Cosmo Carmisciano. 3-d density structure and geological evolution of stromboli volcano (aeolian islands, italy) inferred from land-based and sea-surface gravity data. *Journal of volcanology and geothermal research*, 273:58–69, 2014.

[3] Albert. Tarantola. *Inverse Problem Theory and Methods for Model Parameter Estimation.* Society for Industrial and Applied Mathematics, 2005.

[4] Cédric Travelletti. UnInverse. `https://github.com/CedricTravelletti/UnInverse`, 2022.

Short biography – Cédric Travelletti obtained his MSc in Physics from ETH Zürich in 2016. He then worked in the insurance and banking industry before joining the research group of David Ginsbourger at the University of Bern as a Ph.D. student in November 2018. His thesis focuses on stochastic approaches to estimate implicit sets under indirect measurements. This thesis in funded by the Swiss National Science Fundation under project nr. 178858.