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# Supervised learning and Monte Carlo Markov Chain methods for inverse problem resolution in random neutronics

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**Abstract:**

Neutron noise analysis describes a set of techniques used in nuclear safeguard which aims at identifying a nuclear material by measuring the multiplicities of the detected neutrons [6]. Measurements of the moments of the detection statistics can be used to evaluate nuclear parameters such as the prompt multiplication factor $k_p$, which describes the multiplication rate of neutrons in a fissile medium and is a key value to monitor in nuclear safeguard.

For that purpose, it is required to solve an inverse problem. The common practice is to solve the inverse problem using an analytical approximation of the physical phenomena known as the point model [1, 2]. This method does provide an estimation for the nuclear parameters of interest but the uncertainty quantification is often non-existent. Yet this problematic is critical for this neutron noise analysis since the moments of the distribution are often difficult to measure and their estimations are very noisy. Besides, the strong physical assumptions of the point model induce a systematic bias which is not quantified. Consequently, our goal is to provide methods to quantify the variance of predictions and reduce as much as possible their systematic bias.

In that perspective, a Bayesian approach is used. The objective is to estimate the posterior distribution of the nuclear parameters for a given correlation measurement and to develop efficient ways to sample this distribution. For this specific case, due to the strong non-linearity in the analytical model, the target posterior distribution to be evaluated is very degenerate and thus difficult to sample properly. In a first approach, the analytical model is used in combination with Monte-Carlo Markov chain methods [7, 4], specifically suited to the degenerate probability distributions considered in this problem [3].

Then supervised learning techniques are considered to generalize the analytical model. The two main objectives are to reduce the systematic bias of the simplified analytical model, and to quantify the remaining model uncertainties. Since the goal is to combine these learning techniques with MCMC sampling algorithms, the supervised learning methods used must provide a way to estimate the covariance of the predictions. The predicted covariances can then be used in conjunction with MCMC to account for model bias.

Two main supervised learning methods are considered. At first, Gaussian processes regression for multiple correlated outputs is investigated [8]. Then Bayesian Neural Networks are considered as a second method [5]. Both these supervised learning approaches provide a way to quantify the uncertainties on the predictions and are thus suited to the problematic.

From the early results obtained, the combination of MCMC and supervised learning techniques provide a way to solve the inverse problem while quantifying the confidence in the predictions. These methods also improve significantly the traditional analytical model.

## References


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**Short biography** – After a master degree at Ecole polytechnique and a MsC in Nuclear Engineering at EPFL, I started a PhD at Ecole polytechnique under the supervision of Josselin Garnier. The PhD takes place in collaboration with CEA. The objective of the PhD is to develop mathematical methods for the quantification of uncertainty in neutron noise analysis techniques as well as using supervised learning methods to improve existing analytical models for neutron correlation predictions.