MASCOT-NUM 2022
Clermont Ferrand, France
7-9 June 2022

Supervised deep learning for stochastic lid-driven cavity flows

F. Musco
University of Stuttgart

Supervisor(s): Prof. Dr. A. Barth (University of Stuttgart)

PhD expected duration: Oct. 2021 - Sep. 2024

Address: Allmandring 5b, 70569 Stuttgart, Germany

E-mail: fabio.musco@mathematik.uni-stuttgart.de

Abstract:

We consider a stochastic setup of a benchmark application of the Navier-Stokes equations in two dimensions – the so-called lid-driven cavity flow. Slightly more detailed, we consider an incompressible fluid in a two-dimensional square cavity, that is evenly pushed in one direction at the lid. This pushing then – intuitively – leads to some kind of circular motion of the fluid inside the cavity.

The factors that mainly influence the behavior of the fluid inside the cavity can be divided into two categories. On the one hand, we have fluid specific quantities, such as the density and the viscosity, on the other hand, we have experiment specific quantities, such as the velocity by which the fluid is pushed at the lid – the so-called driving velocity. Our task is to simulate the motion of the fluid inside the cavity for a given set of these quantities.
Oftentimes, when we deal with experiments that occur in nature, uncertainties enter the equations. These uncertainties can arise from inaccurate measurements, or can be caused by the very nature of the experiment itself. In this setup, we choose the driving velocity to be of stochastic nature, namely to be subject to a continuous uniform distribution. This uncertainty in the initial data results in a stochastic problem overall.

In order to gain better insights into the nature of such stochastic setups, for example to quantify uncertainties, one usually relies on repeated sampling from the system. In this particular case, numerical procedures to solve the lid-driven cavity flow are computationally very expensive, making it hardly feasible to repeatedly draw random samples. We address this problem by a supervised deep learning approach with the goal to produce approximations in only a fraction of the time. To resemble the above described stochastic setup, we train a neural network to map a driving velocity to a solution of the corresponding lid-driven cavity flow.

The supervised learning approach still relies on a classical numerical method, since we need to present solutions to the neural network for training. For that, we solve the steady deterministic lid-driven cavity flow in the vorticity-streamfunction formulation by a second-order accurate finite difference method with successive over-relaxation, as proposed by Erturk [1]. We give a brief overview of the numerics involved, and point out some advantages, as well as difficulties, arising from the vorticity-streamfunction formulation of the Navier-Stokes equations.

It turns out that a rather simple architecture of a feedforward deep neural network with just two hidden layers is able to learn the streamfunctions – as a function of the driving velocity – up to a very high accuracy. One of the biggest advantages is, that the neural network achieves quite a good performance, even when it is trained on very few data points. Since the training data is produced by the computationally slow numerical procedure, this fact is very much in line with our goal to speed up computing times.
For example, a network that is trained on 10 data points for driving velocities in the interval $[0.25, 5]$, and a constant kinematic viscosity of 10^{-2}, achieved an average mean squared error of $\sim 2.7 \cdot 10^{-6}$ over 500 samples (490 of which were not presented to the neural network) in the same interval. Furthermore, the neural network is even able to reliably predict streamfunctions for unseen data exceeding the domain of the training data. The highest value for a driving velocity presented to the neural network is 5, and we present predictions for driving velocities up to 7.5 with reasonable accuracies.

In the context of uncertainty quantification, we apply the advantages of fast predictions by a trained neural network to the Monte Carlo method. This approach is very useful since we can estimate the expectation of the system by averaging a large number of pathwise solutions, approximated by the neural network, within a short time, while requiring very little solutions produced by the numerical method to train the network.

References

[1] E. Erturk. Discussions on driven cavity flow. *International Journal for Numerical Methods in Fluids*, 60:275–294, 2009.

[2] F. Musco. *Supervised deep learning for stochastic lid-driven cavity flow.* Master's thesis, 2021.

Short biography – My name is Fabio Musco, and I am a PhD student at the University of Stuttgart since October, 2021. I studied mathematics at the University of Stuttgart (B.Sc. and M.Sc.), and specialized in the fields of stochastics and statistics. In my work, funded by the University of Stuttgart, I research the application of Machine Learning (especially supervised and unsupervised deep learning) in stochastic partial differential equations. The main goal is to gain computationally faster and/or better insights into solutions and quantify uncertainties arising in this context.