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A SUR version of the Bichon criterion for inversion

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Abstract:

Nowadays, many industrial issues are related to a problem of excursion set estimation. This method consists in finding all sets of input parameters of a model such that a quantity of interest built from its outputs remains in a certain area, for example below a fixed threshold. In general, the quantity of interest is an output of a simulator, costly computational, which can be considered as a black-box function. The problem is also known as an inversion problem [3]. For example, the application to a vehicle pollution control system, allowing compliance with pollutant emissions norms can be mentionned [5].

An effective way to solve this kind of problem is to model the costly black-box function of interest as a realization of a Gaussian process. This surrogate model is built thanks to a sequential Design of Experiments (DoE), whose points are chosen accordingly to the optimization of an acquisition criterion (see for example [1] and [8]). Acquisition criteria suitable for inversion include: the deviation number [4], the Bichon criterion also known as Expected Feasibility Function [2], and the Ranjan criterion [9]. These two last criteria are similar adaptations for excursion set estimation of the classical optimization oriented Expected Improvement criterion [6].

In addition, there is a more elaborate and in general more efficient class of criteria that anticipates the impact of adding new points to the DoE: the SUR strategies, for Stepwise Uncertainty Reduction (see [1]). For example, the SUR strategy coupled to the Vorob'ev's theory [7] introduced in [3], is particularly suitable for the inversion framework. Other SUR strategies adapted to inversion can be cited, such as the one associated with the variance of the volume of the excursion set and the one associated with the coverage function (see [1]).

It is shown in [1] that SUR criteria provide better performances compared to direct criteria. However, in the particular case of disconnected excursion sets, the SUR Vorob'ev criterion is not robust against extreme cases, and uses some approximations about the Vorob'ev threshold. The main objective of this work is to propose a SUR version of the Bichon criterion, in order to obtain a method that is easier to set up and more robust than the SUR Vorob'ev method, and to highlight the performances of this new criterion on analytical examples. For example, in the case of the Branin-rescaled function with a threshold equal to 10, it can be seen (Figure 1 at left) that the SUR Bichon criterion is more robust than the SUR Vorob'ev one. Specifically, in pathological cases, the SUR Vorob'ev criterion misses one connected component of the excursion set (Figure 1 at right).

The thesis of Reda El Amri [5] made it possible to implement a methodology for solving an inversion problem under uncertainty, in the case where the output is univariate, the uncertainties are known from a given sample, and the operator considered with respect to the uncertainty is the expectation. The future prospects of our work are to reduce the calculation costs necessary for the inversion and to extend the methodology proposed by Reda El Amri by trying to dispel the different assumptions made.

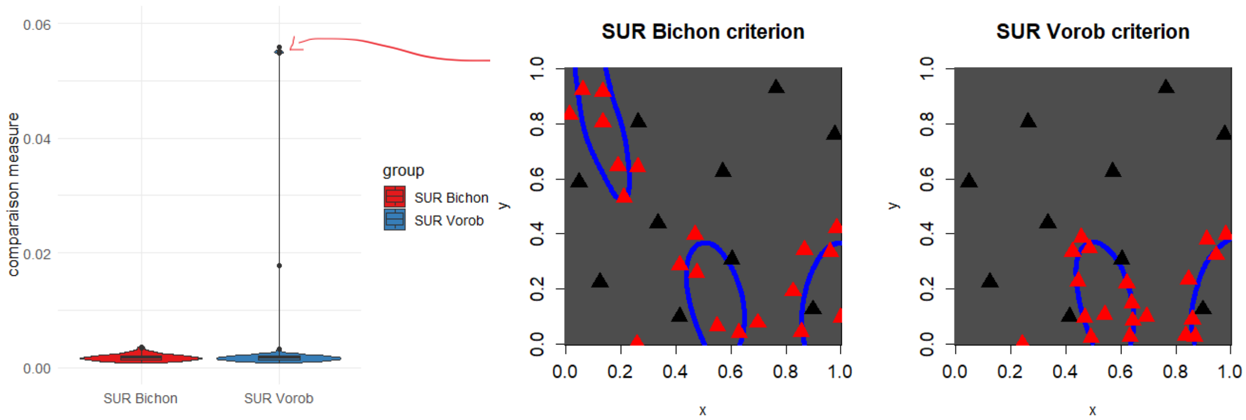

Figure 1: At left, Violinplots of the performance comparaison measure after 20 iterations, for the inversion of the Branin-rescaled function with $T = 10$, for 100 different initial LHS Maximin DoE of 10 points. At right, excursion set estimators (in blue) for the two SUR methods and associated with one DOE (indicated by the red arrow).

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Short biography – I'm a second year PhD student in the AIRSEA team of the Laboratoire Jean Kuntzmann of the Université Grenoble Alpes. My thesis project initiated in October 2020 is on robust inversion with functional data and an application to the design of wind turbines. This project is funded by INRIA, but it is part of a collaboration between IFPEN and INRIA. Before this project, I did a master's degree on Applied Mathematics and Statistics in Université Claude Bernard Lyon I.