MASCOT-NUM 2022
Clermont Ferrand, France
7-9 June 2022

Uncertainty Quantification and Global Sensitivity Analysis of seismic fragility curves using kriging

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PhD expected duration: Nov. 2019 - Nov. 2022

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Abstract:

Seismic fragility curves: The estimation of so-called fragility curves is a crucial part of seismic probabilistic risk assessment (SPRA) or probabilistic based earthquake engineering (PBEE). The fragility curve is the probability of failure of the structure conditionally to a seismic intensity measure, a scalar value indicating the strength of the seismic ground motion, and a vector of mechanical parameters of the structure. Nowadays seismic fragility curve estimation is usually performed by numerical simulations, The computational burden is thus significant, as it requires a consequent number of numerical simulations to provide an accurate estimation. The fragility curve can be expressed as

$$\Psi(a, \mathbf{x}) = \mathbb{P}(z(A, \mathbf{X}) > C | A = a, \mathbf{X} = \mathbf{x}) \ , \tag{1}$$

where A is the random variable of the seismic instensity measure, $\mathbf{X}$ is the random vector of the mechanical parameters of the structure, $z(a, \mathbf{x})$ is a scalar mechanical demand parameter obtained through numerical simulation of the structure with mechanical parameters $\mathbf{x}$ subjected to a seismic ground motion of intensity a. C is a threshold of acceptable behaviour of the structure regarding the mechanical demand parameter studied. Moreover, following the lines of [2], it is of paramount importance to identify which source of uncertainty (coined as epistemic uncertainty) can be reduced in a near-term and regarding a reasonable budget and the source of natural randomness due to a physical phenomena (coined as aleatory uncertainty). Again according to [2], the distinction between aleatory and epistemic uncertainty is a pragmatic way to distinguish which uncertainties the engineers can reduce and those for which it is impossible. It thus allows to make a information based design choice for the engineer. In our case, the aleatory uncertainty is the seismic ground motion uncertainty while the epistemic uncertainty concerns the uncertainty on the mechanical parameters of the structure. It thus of particular interest for the engineer to study the probability distribution of the random functions $a \to \Psi(a, \mathbf{X})$.

Kriging: The goal of kriging is to provide an estimation of the regression function in supervised learning as well as an uncertainty on its prediction. This prediction uncertainty can be interpreted as a degree of confidence on the engineer's quantity of interest, which make this regression technique very appealing for industrial risk assessment studies. Thus, we propose to estimate seismic fragility curves using kriging in order to tackle the computational burden of mechanical simulation and to provide a confidence interval on the estimated fragility curves in the same time. We define the following nonparametric regression model:

$$y(a, \mathbf{x}) = g(a, \mathbf{x}) + \varepsilon \ , \tag{2}$$

where $y(a, \mathbf{x}) = \log(z(a, \mathbf{x}))$ and $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. The logarithm transformation is due to a multiplicative noise in our data. We then suppose that the regression function g is a realization of a real-valued Gaussian process G. Denote that $Y(a, \mathbf{x}) = G(a, \mathbf{x}) + \varepsilon$ is also a Gaussian process. Given a dataset of observations $\mathcal{D}_n = ((a_i, \mathbf{x}_i), y(a_i, \mathbf{x}_i))_{1 \le i \le n}$ and thanks to the kriging equations we can write that $(Y(a, \mathbf{x}) | \mathcal{D}_n) \sim \mathcal{N}(\widehat{G}_n(a, \mathbf{x}), \widehat{\sigma}_n(a, \mathbf{x})^2)$, the kriging based estimator of the fragility curve then writes:

$$\Psi^{(1)}(a, \mathbf{x}) = \Phi\left(\frac{\widehat{G}_n(a, \mathbf{x}) - \log(C)}{\widehat{\sigma}_n(a, \mathbf{x})}\right) \ , \tag{3}$$

where Φ is the cdf of the standard Gaussian distribution. The kriging uncertainty can be propagated by conditioning the fragility curve estimation to a realization G_n of $(G|\mathcal{D}_n)$:

$$\Psi^{(2)}(a,\mathbf{x}) = \Phi\left(\frac{G_n(a,\mathbf{x}) - \log(C)}{\sigma_\varepsilon}\right) , \tag{4}$$

remark that $\Psi^{(1)} = \mathbb{E}_{G_n}[\Psi^{(2)}]$. $\Psi^{(2)}$ is the propagation of the kriging posterior distribution $(G|\mathcal{D}_n)$ in the fragility curve while $\Psi^{(1)}$ is the posterior mean estimator of the fragility curve.

Sensitivity analysis: The goal of sensitivity analysis is to measure the impact of the input parameter uncertainty of a system into its output uncertainty. In particular, GSA takes into account the overall uncertainty ranges on the input parameters. It is natural to perform sensitivity analysis on the input parameters tainted by epistemic uncertainties, in our case the mechanical parameters of the structure, it will help engineers to make design choices affecting the input parameters uncertainties they can master in a near-term. The sensitivity indices will be defined on the seismic fragility curve, seen as a goal-oriented quantity of interest. We propose to estimate the aggregated Sobol indices [3], a natural extension to functional output of the classical Sobol indices for scalar output. Considering that $\mathbf{X} = (X^{(1)}, \ldots, X^{(d)})$, $\bar{\Psi} = \mathbb{E}_{\mathbf{X}}[\Psi(a,\mathbf{X})]$ and $\Psi(a, X^{(i)}) = \mathbb{E}_{\mathbf{X}}[\Psi(a,\mathbf{X})|X^{(i)}]$, the aggregated first order Sobol indices writes:

$$S_i^{FC} = \frac{\mathbb{E}_{X^{(i)}}[\|\bar{\Psi} - \Psi(.,X^{(i)})\|^2]}{\mathbb{E}_{\mathbf{X}}[\|\bar{\Psi} - \Psi(.,\mathbf{X})\|^2]} , \tag{5}$$

where $\|\bar{\Psi} - \Psi(.,X^{(i)})\|^2 = \int_{a_0}^{a_1}(\bar{\Psi}(a) - \Psi(a,X^{(i)}))^2 da$. Moreover, we propose to estimate the β^k indices [1], which are Sobol indices on the kernel embedding of the quantity of interest. These last indices are appealing because kernel methods main purpose is to handle complex data types, such as functional data.

Kriging-based sensitivity indices estimation: Estimation of Sobol indices are usually performed using the so-called pick-freeze method, which require a consequent Monte-Carlo sample size for an accurate estimation (at least one thousand times the input parameter space dimension). We will thus use the kriging metamodel developed for uncertainty quantification of the seismic fragility curves to perform aggregated Sobol and β^k indices estimation. We will also propagate the kriging prediction uncertainty into the sensitivity indices estimates as proposed in [4]. Sampling predictions in the conditional distribution of the kriging metamodel is computationally challenging for this sample size and requires specific algorithms. Monte-Carlo sample size and training size of the kriging model will be chosen so that the uncertainty coming from the Monte-Carlo estimation of the sensitivity indices and from the kriging metamodel are of the same order of magnitude.

References

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Short biography – Clément Gauchy is a PhD Student in EMSI laboratory at CEA Saclay. His PhD is funded by the SEISM institute and in collaboration with École Polytechnique