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Shapley effect estimation in reliability-oriented sensitivity analysis with correlated inputs by importance sampling

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Abstract:

Many physical systems are schematically described by a relation of the form $Y = \phi(\mathbf{X})$, where the d-dimensional input vector $\mathbf{X}$ is random and where the output Y is determined through the deterministic function ϕ. A common application is the analysis of a black box model : ϕ represents a numerical code and $\mathbf{X}$ can be regarded as the external conditions in which the calculation is done. A finite element code in structure engineering is a common example of a such model, whose complexity makes impossible to study it analytically. Moreover, calls to the code are supposed to be expensive and can therefore only be made in limited number.

For safety and/or certification reasons, the **reliability analysis** of the system is a crucial step. Without loss of generality, the failure event can be described as a threshold exceedance event $\{Y > t\}$ where $t \in \mathbb{R}$. It is generally a rare event and it is essential to estimate accurately its probability. Crude Monte Carlo sampling techniques are not adapted to a such estimation when the failure probability is low because their convergence requires too many calls to the function ϕ. More efficient methods such as subset sampling [3] or importance sampling [2] have been developed and allow to globally master this issue.

Moreover, another major step of the study of a computer model is **sensitivity analysis**, which consists in studying the influence of each input component of $\mathbf{X}$ on the variability of the output Y, for example in order to reduce the problem dimension by fixing non-influential components to nominal values. There are many local and global sensitivity analysis tools, including the well-known Sobol indices. Sobol indices [8] are global sensitivity analysis indices which allow to evaluate quantitatively the influence of each input variable on the variability of the output. However, these indices are no longer adapted when the input variables are correlated : even if it is still possible to compute them, they do not allow anymore to clearly identify the origin of the variability of the output. To address this issue, by analogy with game theory [7], it has recently been proposed to consider the Shapley effects for global sensitivity analysis [6]. Nevertheless, their estimation is difficult since naive methods require a high computational effort. However, recent improvements have led to a drastic reduction of the computational effort thanks to estimators requiring only a unique i.i.d. input/output N-sample distributed according to the input distribution [1].

We aim here to combine both reliability and sensitivity analyses in order to perform a **target reliability-oriented sensitivity analysis** [5] of the system, which consists in studying the influence of each input variable on the occurrence of the failure of the system, when the input variables are correlated. It is a challenging task because it aims to obtain reliability-oriented sensitivity analysis results while minimising the computational cost after the estimation of the failure probability. The estimation of the target Shapley effects, i.e. the Shapley effects applied on the quantity of interest $\mathbf{1}(\phi(\mathbf{X}) > t)$, seems to be interesting because it allows to identify the most influential components of $\mathbf{X}$ on the occurrence of the failure of the system. These indices and first estimation schemes have been introduced recently in [4]. Estimators requiring only a unique i.i.d. input/output N-sample distributed according to the input distribution are also proposed, which enable to estimate the target Shapley effects without additional

calls to the function after the estimation of the failure probability by Monte Carlo. Nevertheless, these existing estimators are not adapted when the failure probability is low because their convergence requires a too high computational effort since they are based on a Monte Carlo sampling according to the input distribution.

In this communication, we are introducing new importance-sampling-based estimators of the target Shapley effects which are more efficient than the existing ones when the failure probability is low. Importance sampling is a very well-known method in reliability analysis for estimating a low failure probability more efficiently. In the same way, the principle here consists in rewriting the target Shapley effects according to an auxiliary sampling distribution. Then, the corresponding new estimators require samples drawn according to the importance sampling auxiliary distribution, and lead to a significant variance reduction when the auxiliary distribution is adapted to the problem. Moreover, we also introduce importance-sampling-based estimators requiring only a unique i.i.d. input/output N-sample distributed according to the importance sampling auxiliary distribution, which enable to estimate efficiently the target Shapley effects when the failure probability is low without additional calls to the function after the estimation of the failure probability by importance sampling. In addition, we prove theoretically that using the optimal auxiliary distribution for estimating a failure probability by importance sampling, which is the input distribution restricted to the failure domain, as the auxiliary distribution improves the estimation of the target Shapley effects in comparison to the existing estimators. Recalling that if the reliability analysis has been done efficiently by importance sampling, the samples should be drawn according to an auxiliary distribution close to the optimal one, the latest result justify then that it is numerically beneficial to reuse the available sample to estimate the target Shapley effects with the latest less expensive estimators. Finally, we illustrate the practical interest of the proposed estimators on the Gaussian linear case and on two more complicated real physical examples.

References

[1] Baptiste Broto, François Bachoc, and Marine Depecker. Variance reduction for estimation of Shapley effects and adaptation to unknown input distribution. *SIAM/ASA Journal on Uncertainty Quantification*, 8(2):693–716, 2020.

[2] James Bucklew. *Introduction to rare event simulation*. Springer Science & Business Media, 2004.

[3] Frédéric Cérou, Pierre Del Moral, Teddy Furon, and Arnaud Guyader. Sequential Monte Carlo for rare event estimation. *Statistics and computing*, 22(3):795–808, 2012.

[4] Marouane Il Idrissi, Vincent Chabridon, and Bertrand Iooss. Developments and applications of Shapley effects to reliability-oriented sensitivity analysis with correlated inputs. *Environmental Modelling & Software*, 143:105115, 2021.

[5] Amandine Marrel and Vincent Chabridon. Statistical developments for target and conditional sensitivity analysis: Application on safety studies for nuclear reactor. *Reliability Engineering & System Safety*, 214:107711, 2021.

[6] Art B Owen. Sobol'indices and Shapley value. *SIAM/ASA Journal on Uncertainty Quantification*, 2(1):245–251, 2014.

[7] L. S. Shapley. A value for n-person games. *Contributions to the Theory of Games*, (28):307–317, 1953.

[8] Ilya M Sobol. Sensitivity analysis for non-linear mathematical models. *Mathematical modelling and computational experiment*, 1:407–414, 1993.

Short biography – I graduated in 2021 from the graduate engineering school ISAE-SUPAERO in Toulouse and I also obtained a MSc degree in applied mathematics from Toulouse III - Paul Sabatier University. Then, I performed my final-year internship at *ONERA* in Toulouse which led to my current PhD thesis co-funded by *ONERA* and *Toulouse III - Paul Sabatier University*. The main goal of the PhD thesis is the estimation of the Shapley effects for reliability-oriented sensitivity analysis.